MATH 425b ASSIGNMENT 1
SPRING 2009
Prof. Alexander
Due Monday January 26.

Rudin Chapter 7 #1, 2, 3, 6, ,8, 9 and:

(I) Let $f_n(x) = nxe^{-nx^2},\ 0 \leq x \leq 1$.
(a) Show that f_n converges uniformly on $[0, 1]$ to some f, and find f.
(b) Show that $\int_0^1 f_n(x)\, dx \not\to \int_0^1 f(x)\, dx$.

(II) Suppose $f_n : E \to \mathbb{R}$ is uniformly continuous for all n, and $f_n \to f$ uniformly. Show that f is uniformly continuous.

(III) Define the functions $f_n : (-\pi, 3\pi) \to \mathbb{R}$ by

$$f_n(x) = \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots \pm \frac{\sin nx}{n}.$$

It can be shown (you don't need to prove it!) that this sequence of functions converges pointwise to

$$f(x) = \begin{cases} \frac{x}{2}, & x \in (-\pi, \pi), \\ 0, & x = \pi \\ \frac{x}{2} - \pi, & x \in (\pi, 3\pi). \end{cases}$$

Show that this convergence cannot possibly be uniform.

HINTS:

(1) Uniformly convergent $\iff$ uniformly Cauchy.

(2) Use

$$f_n g_n - fg = f_n g_n - f g_n + f g_n - fg.$$

(3) Because of Problem 2, the functions you choose must be unbounded. You can find examples with $f_n = g_n$.

(6) Split the series into two separate ones with numerators x^2 and n.

(8) This is a fairly quick one, if you apply the right theorem from Ch. 7.

(9) Apply the triangle inequality to $|f_n(x_n) - f(x)|$.

(II) Theorems 7.11 and 7.12 establish the same result with "uniformly continuous" replaced by just "continuous." The proof here is similar.

(III) Don't try to do it using epsilons and deltas! Find a theorem you can cite. Sketch the graph of f.