

MATH 425b ASSIGNMENT 2
SPRING 2008
Prof. Alexander
Due Wednesday February 4.

Chapter 7 #16, 18 and:

(I) Suppose \mathcal{F} is an equicontinuous subset of $C[a, b]$. Show that $\overline{\mathcal{F}}$ (the closure in the uniform metric) is equicontinuous.

(II) Let $\alpha > 0$ and define

$$E = \{f \in C[0, 1] : f(0) = 0, |f(y) - f(x)| \leq |y - x|^\alpha \text{ for all } x, y\}.$$

(a) Show that E is equicontinuous.

(b) Show that E is a closed subset of $C[0, 1]$ (with the uniform metric, as always.)

(III)(a) Let $[0, A]$ be an interval and P a polynomial. Show that for every $\epsilon > 0$ there is a polynomial Q with rational coefficients such that $\|P - Q\|_\infty < \epsilon$, on $[0, A]$.

(b) Show that $C[0, A]$ has a countable subset which is dense.

(IV) Let $\{f_1, \dots, f_n\}$ be a fixed set of continuous functions on $[a, b]$. Let \mathcal{F} be the class of all functions of the form $\sum_{i=1}^n c_i f_i$ with $|c_i| \leq 1$ for all i . Show that \mathcal{F} is equicontinuous.

(V) For this problem, $\|f\|_\infty$ means $\sup_{x \in \mathbb{R}} |f(x)|$, i.e. the sup is over all of \mathbb{R} .

(a) Show that if P is a nonconstant polynomial then $\|P\|_\infty = \infty$.

(b) Show that if a function $f : \mathbb{R} \rightarrow \mathbb{R}$ is a uniform limit of polynomials (that is, $\|P_n - f\|_\infty \rightarrow 0$), then f itself is a polynomial.

HINTS: (Note (I), (II), (V) should be easier than (III), (IV).)

(16) Mimic parts of the proof of Theorem 7.25.

(I) Show that if some δ works for all $f \in \mathcal{F}$, then this same δ works for limit points (in the uniform metric) of \mathcal{F} .

(II)(a) Use the definition directly.

(III) For (b), use (a).

(V)(a) If P has degree k , consider $P(x)/x^k$ as $x \rightarrow \infty$.

(b) Consider $P_n - P_m$. Note if f is a polynomial then taking $P_n = f$ for all n is allowed.