

MATH 425b ASSIGNMENT 7 (Revised)
SPRING 2009
Prof. Alexander
Due Monday April 6.

Rudin Chapter 9 #26, 29 and:

(A) This problem tests your understanding of the definition of derivative. A system has 3 inputs x_1, x_2, x_3 , and the corresponding output $f(\mathbf{x})$ has 3 components; more precisely $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is \mathcal{C}' . The function f is not known to the researcher, but he can input values \mathbf{x} and observe the output values $f(\mathbf{x})$. So far he has the following observations:

$$\begin{aligned}f(2, 3, 4) &= (7, 6, 5) \\f(2.01, 3, 4) &= (6.99, 6.03, 5.04) \\f(2.01, 3.01, 4) &= (7.01, 6.06, 5.05) \\f(2.01, 3.01, 4.01) &= (7.01, 6.02, 5)\end{aligned}$$

Use this information to find the approximate value of the derivative $f'(2, 3, 4)$ (expressed as a matrix) and to estimate $f(2, 3.01, 4.01)$.

(B) Let U be a connected open subset of \mathbb{R}^2 and let $f : U \rightarrow \mathbb{R}$ be a \mathcal{C}' function with $\frac{\partial f}{\partial y} = 0$ everywhere in U .

(a) Recall that U is called *convex* if for any two points in U , the line segment connected these two points is contained in U . If U is convex, show that f does not depend on y , that is, $f(x, y) = g(x)$ for all x, y .

(b) In contrast to (a), find an example of a non-convex U and an f as above, in which f does depend on y .

(C) Let $E \subset \mathbb{R}^2$ be open and let $f : E \rightarrow \mathbb{R}$. Suppose that D_1f exists everywhere in E and is continuous at some point $\mathbf{a} \in E$, while D_2f exists only at \mathbf{a} . Show that f is differentiable at \mathbf{a} . HINT: Express $f(\mathbf{a} + \mathbf{h}) - f(\mathbf{a})$ as a sum of two increments. Also, if $f'(\mathbf{a})$ does exist its matrix must have entries $D_1f(\mathbf{a}), D_2f(\mathbf{a})$ so you just have to show that the transformation with this matrix satisfies the definition of derivative.

HINTS:

(B)(b) This is a little tricky—you have to think geometrically about the graph of such an f . Use a set U which is actually “U” shaped (not necessarily opening upward, though.) To build your f , you may want to make use of functions on \mathbb{R} which are \mathcal{C}' and not constant, but are constant on an interval, for example,

$$h(t) = \begin{cases} 0, & t \leq 0, \\ t^2, & t > 0. \end{cases}$$