MATH 425b ASSIGNMENT 9
SPRING 2009
Prof. Alexander
Due Friday May 1.

Rudin Chapter 10 #16, 20, 21abd, 24, 25 and:

(I) A differential equation of the form

$$M(x,y) + N(x,y)\frac{dy}{dx} = 0, \qquad (x,y) \in \mathbb{R}^2, \quad M,N \in \mathcal{C}''$$

is called *exact* if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Show that there then exists an $F(x,y)$ such that $\frac{\partial N}{\partial x} = M$, $\frac{\partial F}{\partial y} = N$, so the equation can be rewritten

$$\frac{d}{dx}F(x,y) = 0.$$

Notice this is not a partial derivative—instead think of $y = y(x)$ as a function of x and use the chain rule.

(II) Let S be the portion of the surface $z = x^4 + y^2$ (in $\mathbb{R}^3$) which lies above $[0,1]^2$ and let

$$\omega = x\ dy \wedge dz + y\ dz \wedge dx + z\ dx \wedge dy.$$

Find a natural parametrization Φ of S and calculate $\int_\Phi \omega$.

(III) Let S be the positively oriented boundary of the set $E \subset \mathbb{R}^3$ enclosed by $y = x^2, y = 4, z = 0$ and $z = 1$. Let

$$\omega = xyz\ dy \wedge dz + (x^2 + y^2 + z^2)\ dz \wedge dx + (x + y + z)\ dx \wedge dy.$$

Calculate $\int_S \omega$.

HINTS:

(16) When you write out an expression for $\partial^2\sigma$, what is the coefficient of σ_{kl} for a fixed k,l?

(21)(d) Just calculate $d(\arctan \frac{y}{x})$.

(25) To get $f(\mathbf{x})$ in problem 24, you do a line integral along the line segment from $\mathbf{p}$ to $\mathbf{x}$. But does it really matter what $\mathcal{C}'$ curve you use, from $\mathbf{p}$ to $\mathbf{x}$?

(I) Consider the form $M\ dx + N\ dy$.