MATH 525b ASSIGNMENT 1
SPRING 2009
Prof. Alexander
Due Monday January 26.

Chapter 5 #30, 32, 38, 39 and:

(I) Let $E_1, E_2, \ldots$ be disjoint closed sets in $[0,1]$. Show that $\cup_{n\geq 1} E_n$ is not all of $[0,1]$.

(II) Here is a review problem on dual spaces. Let X be a normed linear space and suppose the dual X^* is separable. Show that X is separable.

HINTS:

(32) Make use of the identity map on $\mathcal{X}$, which you can view as a mapping between two Banach spaces.

(I) First note that the property of being n.d. (= "nowhere dense") depends on the space, that is, we can have $E \subset Y \subset X$ with E n.d. in X but not n.d. in Y.

The sets E_n certainly don't have to be n.d. in $[0,1]$—they could have nonempty interiors, so let us throw out those interiors first. Let $F_n = E_n \backslash E_n^o$ (where E_n^o = interior) and $F = \cup_n F_n$. Show that F_n is n.d. in $[0,1]$, and then that F_n is n.d. in F.

(II) Let $\{f_n\}$ be a countable dense subset of X^* and for each n let x_n satisfy $||x_n|| = 1$ and $|f_n(x_n)| \geq \frac{1}{2}||f_n||$. Let S be the subspace spanned by $\{x_n\}$, that is, the set of all *finite* (not infinite!) linear combinations of vectors x_n. Show that S is dense.