

MATH 525b ASSIGNMENT 2
SPRING 2009
Prof. Alexander
Due Wednesday February 11.

Chapter 4 #5, 16, 17, 34, 54, 58 and:

(A) Let X be a σ -compact LCH space. Let $\Phi(t) = t/(1+t), t \geq 0$, and for $f, g \in C(X)$,

$$\rho(f, g) = \sum_{n=1}^{\infty} 2^{-n} \Phi \left(\sup_{x \in \overline{U}_n} |f(x) - g(x)| \right),$$

where U_1, U_2, \dots are as in Prop. 4.39. Show that $f_n \rightarrow f$ uniformly on compact sets if and only if $\rho(f_n, f) \rightarrow 0$. It can be shown that ρ is a metric, but you need not do this.

HINTS:

All problems: Be careful that you don't use arguments and deductions that would be valid for Euclidean space, or metric spaces, but not necessarily for more general topological spaces.

(17) In the definition of Hausdorff (same as T_2 , on p. 116), you can take U, V from a base for \mathcal{T} (why?)

(34) Use a base for the topology.

(54)(b) For the "but..." part, you need to find an infinite compact subset of \mathbb{Q} .

(58) If a set $E \subset \prod_{\alpha \in A} X_\alpha$ contains an open set, what special type of open set must it also contain?

(A) Show that K compact implies $K \subset U_n$ for some n .