Modeling Resonances with Phase Modulated Self-Similar Processes

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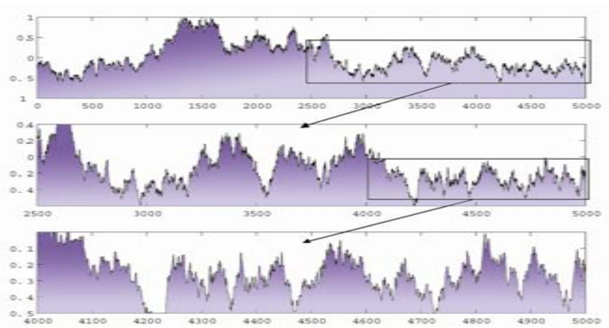
Self-Similar Stochastic Processes

A process is called self-similar if its statistical properties remain invariant under time rescaling:

$$X(rt) \overset{d}{=} r^H X(t)$$



Self-similarity often implies 1/f power spectra and heavy trailed distributions.

Used for modeling an incredible variety of physical and man-made phenomena like Phase noise in electronic devices, Network traffic, Heartbeat variations, Turbulence, statistical properties of music and many more.

Key observation: In many of these phenomena, there is a fundamental periodic behavior and **the self-similar process acts as a fluctuation in the frequency or the phase of this periodicity**. **Examples**: Heartbeat variations, Phase noise in oscillators and resonance fluctuation in speech turbulence. We need a model where a self-similar process acts as a phase fluctuation (=modulation) signal to a basic periodic behavior.

Proposed model for phase modulated self-similar processes

$$X(t) = A\cos(\omega_c t + \lambda P(t) + \varphi_0)$$

Where P(t) is a self-similar process, A is a constant amplitude, ω_c is the frequency of the periodic signal, λ is a modulation coefficient and φ_0 is a phase offset.

Need a probabilistic model for self-similar stochastic process P(t) to get some results for the statistics of the modulated process X(t).

Popular model: Fractional Brownian Motion (Gaussian, Stationary increments Self-Similar process. (Jargon: **Gaussian SSSI process**)

Generalized model: In many cases the systems demonstrate **impulsiveness** (=very spiky behavior, not modeled by Gaussian distribution). Popular models:Fractional Stable Levy motion, other Levy processes. The process has a **Symmetric alpha-Stable distribution**

(Jargon: **SαS-SSSI process**) (=Very general model, FBM special case)

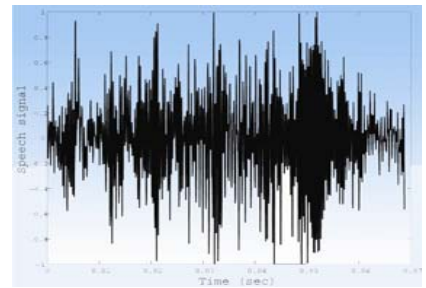
Main Theoretical result: We analytically derive the second order statistics of a X(t), for **a very general family of self-similar processes P(t)** (*any H-SαS-SSSI process*). Specifically, we prove that X(t) is zero mean, WSS process (notice that **P(t) is not stationary**) with autocorrelation function:

$$R_{xx}(\tau) = \frac{1}{2}\cos(\omega_c \tau)\exp(-c\,|\tau|^{\alpha H})$$

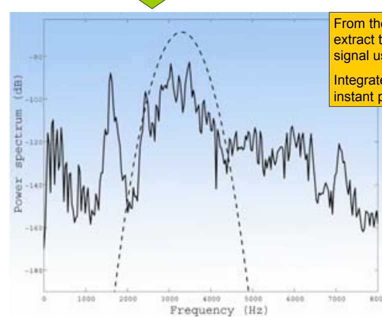
H and α are the parameters of P(t) and c is a function of (λ, α). We further derive explicit formulas for the autocorrelation and power spectrum for the gaussian as a special but interesting case.

Application on fricative Speech Resonances

Pick isolated speech signal for fricative phoneme, (this case /Z/)

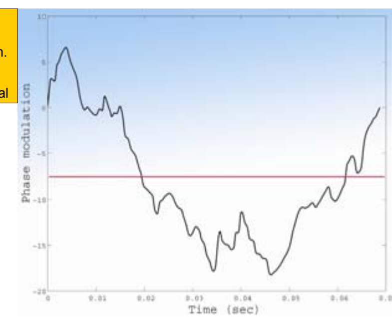


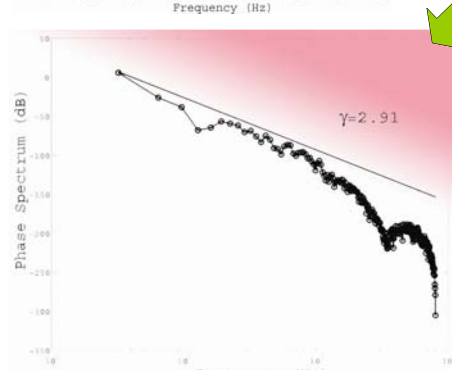
Find spectrum and pick broad resonance (either manually or with iterative ESA scheme). Isolate resonance using a Gabor filter (shown as dotted line)



From the bandpass signal, extract the instant frequency signal using the ESA algorithm.

Integrate the result to get the instant phase modulation signal





This instant phase modulation signal P(t) is modeled as a self-similar process. For this real signal we see that the power spectrum decays as 1/f^γ for a very broad frequency range. Other tests in time domain and wavelet domain have been performed and the model seems to be very suitable for voiced and unvoiced fricatives. The slope parameter γ can be estimated either by least squares or other estimation methods developed for self-similar processes.