

CS303 (Spring 2008) — Assignment 10

Due: 04/09/2008

- (1) This problem merely asks whether you've followed and understood the definitions from class (also given in the online lecture notes).
- (a) What is a “problem”?
 - (b) What do we mean if we say that a program P “decides” a problem?
 - (c) What is a “reduction from a problem X to a problem Y ”?
 - (d) What do we mean when we write $X \leq_m Y$.
 - (e) If we have proved $X \leq_m Y$, what do we know about which of X, Y could be decidable?

- (2) Define the set

$$\text{NONEMPTY} = \{P \mid \text{Program } P \text{ terminates on at least one input}\}.$$

Prove that NONEMPTY is undecidable. You should use a reduction to prove this, even though it would follow easily from Rice's Theorem. (Hint: it will probably help you if you first write down what your reduction needs to do. Also, if you write that down correctly, but can't figure out the actual reduction, you would earn partial credit.)

- (3) Define the set

$$\text{VARIED} = \{P \mid \text{Program } P \text{ outputs different values for at least two inputs}\}.$$

Prove that VARIED is undecidable. Same requirement and hints as for the previous problem.

- (4) Define the following two sets of programs:

$$\begin{aligned} X &= \{P \mid \text{Program } P \text{ terminates on input 42, but on no other inputs}\} \\ Y &= \{P \mid \text{Program } P \text{ terminates for infinitely many inputs}\}. \end{aligned}$$

Prove that $X \leq_m Y$. (This one is quite a bit more tricky than the previous problems.)