

CS 570 (Fall 2007) — Quiz Solutions
08/29/2007

This quiz is closed book, closed notes. For multiple choice questions, more than one answer may be correct. Mark all that apply. There are seven questions on this quiz.

G O O D L U C K

(1) [1 point]

Write your name legibly on this quiz (and any extra sheets you may be submitting).

(2) [10 points]

Prove by induction that the recurrence $T(0) = 1, T(k) = 3T(k-1) - 1$ has the closed-form solution $T(k) = \frac{3^k+1}{2}$ for all k .

(Note: it may be possible to prove this with other techniques. However, only a proof by induction will be accepted here.)

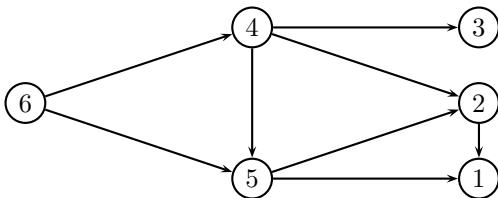
Base case $k = 0$: $T(0) = 1 = \frac{3^0+1}{2}$.

Inductive step from k to $k+1$:

$$T(k+1) = 3T(k) - 1 \stackrel{IH}{=} 3\frac{3^k+1}{2} - 1 = \frac{3^{k+1}+3-2}{2} = \frac{3^{k+1}+1}{2}$$

(3) [10 points]

If you perform a depth-first search (DFS) on the given graph, starting from node 6, and break ties in favor of lower-numbered nodes, in which order will the nodes be visited *for the first time*?



The order is 6-4-2-1-3-5

(4) [9 points]

Give asymptotic bounds for the following sums, in the following style: $\sum_{i=1}^n i = \Theta(n^2)$. If you don't know matching upper and lower bounds, you can get partial credit for good upper (big-O) or lower (big- Ω) bounds.

- (a) $\sum_{i=1}^n \sum_{j=1}^i j: \Theta(n^3)$
- (b) $\sum_{i=1}^n 2^i: \Theta(2^n)$
- (c) $\sum_{i=1}^n \frac{1}{i}: \Theta(\log n)$

(5) [8 points]

You want a data structure that allows you to insert (key, value) pairs into a “database” and look them up by the key (which is an integer number). Assume that your database already contains n items. Give worst-case asymptotic bounds for the insertion and lookup times *for one more item*, for each of the following simple data structures:

	Insertion	Lookup
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$ (binary search)
Balanced Search Tree (e.g., AVL, B-Tree, Red-Black)	$\Theta(\log n)$	$\Theta(\log n)$

(6) [8 points]

- (a) Which of these sorting algorithms have a worst-case running time of $\Omega(n^2)$ — mark all that apply:
 - Insertion Sort
 - Merge Sort
 - Quick Sort (with good median finding)
 - Selection Sort
- (b) When we say that Bubble Sort has worst-case running time $\Omega(n^2)$, what do we mean?
 - There is a constant c such that for all arrays of length n , Bubble Sort takes at least cn^2 steps.
 - There is a constant c such that for all n , there is an array of length n on which Bubble Sort takes at least cn^2 steps.
 - There is a constant c and an n such that for some array of length n , Bubble Sort takes at least cn^2 steps.

(7) [10 points]

Which of the following are *appropriate* behaviors in solving homework?

- Solving the problems myself.
- Discussing solutions without listing collaborators on the writeup.
- Discussing solutions with other students if I list them in the writeup.
- Copying solutions from other students if I list them in the writeup.
- Discussing solutions with people outside the class, including online groups, so long as I list them in the writeup.
- Copying solutions from books, papers, or online archives, so long as I list them in the writeup.