

# Performance Analysis of Mobility-assisted Routing

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## ABSTRACT

Traditionally, ad hoc networks have been viewed as a connected graph over which end-to-end routing paths had to be established. Mobility was considered a necessary evil that invalidates paths and needs to be overcome in an intelligent way to allow for seamless communication between nodes. However, it has recently been recognized that mobility can be turned into a useful ally, by making nodes carry data around the network instead of transmitting them. This model of routing departs from the traditional paradigm and requires new theoretical tools to model its performance. A *mobility-assisted* protocol forwards data only when appropriate relays encounter each other, and thus the time between such encounters, called *hitting* or *meeting* times, are of high importance.

In this paper, we derive accurate closed form expressions for the expected encounter time between different nodes, under commonly used mobility models. We also propose a mobility model that can successfully capture some important real-world mobility characteristics, often ignored in popular mobility models, and calculate hitting times for this model as well. Finally, we show how these hitting time results can be used to analyze the performance of mobility-assisted routing. We demonstrate that derivative results concerning the delay of various routing schemes are very accurate, under all the mobility models examined. Hence, this work helps in better understanding the performance of various approaches in different settings, and can facilitate the design of new, improved protocols.

## 1. INTRODUCTION

Traditionally, ad hoc networks have been viewed as a connected graph over which end-to-end routing paths need to be established to enable data delivery. Mobility of nodes was considered a necessary evil that invalidates existing routing entries over time, and needs to be intelligently overcome to allow for seamless communication between nodes. However, it has recently been recognized that mobility can be transformed into a useful ally and be exploited to improve ad hoc network performance [1, 15].

The seminal paper by Kumar and Gupta [16] showed that the traditional model of ad hoc network routing is not scalable. Maintaining multi-hop paths and transmitting data over these paths results in excessive relay transmis-

sions that dominate the available bandwidth. This quickly diminishes the capacity available per node for end-to-end traffic. To deal with this inherent difficulty, researchers proposed to allow only a fixed number of transmissions per message [15]. A message is handed over to the first few relays encountered, and each relay will then carry the message all the way to its destination itself [12, 15, 31]. This method guarantees a constant per node capacity, regardless of network size.

On a different note, there has been a growing interest in the past few years in applications that can operate over networks that are disconnected most of the time. Such applications often target environments where constant connectivity might be difficult, as is the case for example in deep-space interplanetary networks (IPN [10]), Vehicular Ad hoc Networks (VANET [40]), sensor networks for wildlife tracking [23], etc. Alternatively, operation over disconnected networks may be desirable for economic reasons, as for example in the case of low-cost Internet provision in remote communities [13] or third-world countries [21, 27]. Such networks are often collectively referred to as Delay Tolerant Networks (DTN) [1]. In Delay Tolerant Networks traditional routing approaches fail, because they require the existence of complete end-to-end paths to be able to deliver any data. To overcome this issue, messages need to get carried by mobile nodes between disconnected parts of the network [30, 43]. A message gets forwarded to some other relay encountered, carried further, gets forwarded again, and so on and so forth until it reaches its destination.

What the above two approaches share in common is that node mobility is exploited to carry messages around the network as part of the routing algorithm. We will refer to these schemes collectively as *mobility-assisted routing* (in related literature they are also referred to as *encounter-based* or *store-carry-and-forward*). Mobility-assisted routing consists of a series of *independent* forwarding decisions, that take place when two nodes meet each other, and these nodes are completely oblivious of the specific path the message will end up following. Nodes carry a set of messages, possibly for long periods of time, until other nodes are encountered, exchange messages according to the specific protocol, and continue their trip. This constitutes an important departure from the existing routing paradigm, and thus different theoretical tools and models are necessary to analyze the performance of such schemes.

Since message transmissions in this context occur only when nodes meet each other, *the time elapsed between such meetings is the basic delay component*<sup>1</sup>. Therefore, in order to be able to evaluate the performance of *any* mobility-assisted routing scheme, it is necessary to know

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<sup>1</sup>It is reasonable to assume that message transmission will be significantly faster than node movement for modern wireless devices.

the statistics of encounter times between nodes, called *hitting* or *meeting* times. These are the times until a node, which, say, just received a message, *first* encounters a given other node that can act as a relay. In other words, it essentially is the *minimum time elapsed before the message can be forwarded any further*. These statistics constitute the basic component in the delay expression of any scheme, and they largely vary depending on the specific mobility model in hand. Although a number of efforts have been made towards analyzing the delay of mobility-assisted routing [14, 26, 29, 31, 33, 36], hitting and meeting times have largely remained unstudied for most mobility models. The random walk model is, to the best of our knowledge, the only exception [30, 35, 36]. As a consequence, most existing results regarding the delay of various routing schemes are either only asymptotic [14, 26, 31] or not completely analytical, in that specific parameters of the model need to be acquired from simulations [29, 33]. However, complete analytical expressions are necessary in order to theoretically compare different schemes, and predict their performance in a large range of different settings.

Additionally, the majority of mobility models commonly used in theoretical analysis (or simulation) are not realistic. Mobility models like Random Waypoint [9], Random Direction [5], Random Walk, [2] etc., assume that each node may move equally frequently to every network location. Furthermore, such models usually assume that all nodes have the same mobility characteristics, that is, every node’s mobility process is identical and independently distributed from all others (IID). However, numerous recent studies based on mobility traces from real networks (e.g. university campuses, conferences, etc.) have demonstrated that these two assumptions rarely hold in real-life situations [4, 18–20, 25]. Although there exist some efforts to create more accurate mobility models [3, 22, 37], a lot of these models are quite complicated and/or tailored to specific scenarios, making them difficult to be used in theoretical studies.

In this paper, we analyze the hitting times for a number of popular mobility models like the Random Waypoint and the Random Direction models, and derive accurate closed form expressions for all quantities involved. We show how these expressions can be used in a general analytical framework to calculate the expected delay of various mobility-assisted routing schemes in a “plug n’ calculate” manner. Specifically, we demonstrate that derivative results based on these expressions, and concerning the delay of various routing schemes under various mobility models are also very accurate. Additionally, we propose the *Heterogeneous Community-based Mobility Model*, a novel mobility model that successfully captures important insight regarding real-world mobility, yet is analytically tractable. To demonstrate this, we derive various encounter-based statistics for this model, and show how it can be incorporated in the more general analytical framework, as well.

In the next section we go over some existing related work, and summarize our contributions. In Section 3 we derive theoretical closed form expressions for the hitting times under the Random Waypoint and the Random Direction mobility models. Then, in Section 4 we introduce our own Heterogeneous Community-based model and analyze its encounter-based statistics. Section 5 incorporates the hitting times into the general analytical framework that can be used to predict the performance of mobility-

assisted routing under various mobility models. Finally, Section 6 concludes the paper.

## 2. RELATED WORK

The first paper that proposed the use of mobility to overcome the limited capacity problem of ad hoc networks was the “2-hop relay” scheme of Grossglauser et al. [15]. In this scheme, at most one relay is allowed to carry a copy of the message, and thus the message path is at most two hops long. Although this scheme guarantees a  $\Theta(1)$  capacity per node, it may result in long delivery delays because the relay has to wait until it moves all the way within range of the destination itself.

A significant research thread has spawned thereafter exploring the fundamental trade-offs between the capacity and the delay of the 2-hop and other similar schemes (e.g. [12, 14, 26, 28, 31]). For example, it has been shown that by handing over copies to more than one (yet fixed number of) relays, delay is reduced for most mobility models, while asymptotic capacity (i.e. per node capacity as the number of nodes grows to infinity) remains constant [14].

Despite the amount of existing work, most results are of asymptotic nature. Although asymptotic results provide valuable insight on the scalability of a given family of protocols, explicit results are often necessary to design and compare efficient and practical schemes. Furthermore, the majority of these works are concerned with delay in *connected* networks, where the transmission range of each node has to scale with the number of nodes, in order to ensure connectivity [17]. This makes all such analytical results strictly a function of the number of nodes (e.g. [14, 31]). Here, we’re interested in a much wider range of connectivity scenarios, where transmission range, number of nodes, and network size are *independent* parameters, whose individual effect on performance we would like to quantify.

A significant amount of theoretical work has also recently emerged in the context of intermittently connected or Delay Tolerant Networks [29, 31, 33, 35, 36, 39, 42]. In contrast to connected networks where mobility is only used to reduce the number of transmissions, there mobility is a *necessary component* of the routing functionality. These papers try to analyze the delay of flooding or other schemes that route one or more message copies according to some algorithm, in networks that are not connected for the majority of time. However, in doing so, they often assume that the expected time between encounters is just a parameter of the mobility model that can be acquired from simulations or curve fitting [29, 31, 33]. Although this makes these results quite generic, at the same time it also significantly reduces the usefulness of analytical expressions. Running a simulation every time for the specific network configuration in hand, to estimate a parameter of the theoretical model, comes short of being able to predict and compare the performance of different schemes under a wide range of settings. Explicit expressions for encounter times under a given mobility model are necessary, to derive explicit expressions for the delay of *any* mobility-assisted routing scheme.

To fill this important gap, in this paper we analyze the hitting times for some popular mobility models, like the Random Waypoint [9] and the Random Direction model [5]. Although various statistical properties of the Random Waypoint and the Random Direction model (e.g. node distri-

bution [6], convergence [8, 41], etc.) have been studied, hitting and meeting times under such models are not available. To the best of our knowledge hitting and meeting times have only been analyzed in the past for Random Walk mobility [2, 30, 35]. Also, the work that is closest to ours is probably that of [32]. There, the authors use a similar methodology to the one we’ll use, but only derive upper and lower bounds on the meeting time between two nodes performing Random Waypoint movement, and use it to calculate an asymptotic result. Finally, we also show how hitting times can be integrated with generic routing delay expressions like the ones in [33, 35, 36, 39] into a powerful framework. A framework along these lines was proposed in [29].

A lot of work exists also in the arena of mobility modelling. It has been widely recognized that mobility models commonly used in simulation studies or theoretical analysis, like Random Waypoint, Brownian motion, etc. [11], although easy to use, do not closely resemble mobility involving humans [19, 20, 22]. A large amount of mobility traces from university or conference environments have recently become available and have been extensively analyzed to extract basic characteristics of real-world mobility [4, 18–20, 25]. These traces indicate that, in most of these scenarios nodes do not move randomly around the whole network area. Instead, nodes usually have some locations where they spend a large amount of time (e.g. home, office building, etc.). A few times in the day, they have to commute back and forth from these locations, or might roam around the network for some time (e.g. running errands). Additionally, contrary to the common assumption, node movement is not IID. Different nodes visit different location more often, and some nodes may be more mobile than others.

A number of mobility models have been proposed that better capture specific scenarios, like freeway movement [3], human mobility in campus environments [22, 37], group mobility models [11], etc. The work closest to ours in terms of mobility modelling is probably that of [24]. Although this work manages to capture the locality of node movement, it does not account for heterogeneity in different nodes’ mobility processes. Summarizing, a model that would capture most of the details of real-life mobility, a very challenging endeavor itself, would be extremely difficult to allow any theoretical analysis. For this reason, we propose here a novel mobility model that: (i) successfully captures the basic intuition derived from traces regarding real-world mobility, (ii) is highly tunable to be able to model a large range of scenarios, and (iii) is analytically tractable. We calculate encounter-based statistics under this more realistic mobility model as well, and show how it can be incorporated into the more general framework.

### 3. HITTING AND MEETING TIMES UNDER POPULAR MOBILITY MODELS

In this section we will look into the encounter-based statistics of two commonly used mobility models, namely Random Waypoint (RWP) and Random Direction (RD) mobility. However, before we do so we first introduce some useful definitions and notation and state the assumptions we’ll be making throughout the remaining of the paper.

- (a) All nodes exist in area  $U$  of size  $\|U\| = N$ , and have a transmission range equal to  $K$ . The position of node

**Table 1: Notation**

$ET_{mm}$	expected hitting time under “MM”
$EM_{mm}$	expected meeting time under “MM”
$N$	size of network area
$K$	transmission range
$\bar{L}$	expected epoch length
$\bar{v}$	average node speed
$\bar{T}_{stop}$	average pause time after an epoch
$\bar{T}$	expected epoch duration
$\hat{v}_{mm}$	normalized relative speed under “MM”

$i$  at time  $t$  is denoted as  $X_i(t)$  or  $X_i$  if it is static.

- (b) All the mobility models we deal with are *epoch-based*; An *epoch* is a given period of time during which a node moves towards the same direction and with the same speed; Each node’s trajectory is a sequence of epochs.
- (c) The length  $L$  of an epoch, measured as the distance between the starting and finishing points of it, is a random variable with expected value  $\bar{L}$ .
- (d) The speed of a node during an epoch is randomly chosen from  $[v_{min}, v_{max}]$ , with  $v_{min} > 0$ ,  $v_{max} < \infty$  and average speed  $\bar{v}$ .
- (e) At the end of each epoch a node pauses for a random amount of time chosen from  $[0, T_{max}]$ , with average pause time  $\bar{T}_{stop}$ .
- (f) The expected duration of an epoch (without the pause time) is denoted as  $\bar{T}$ .
- (g) Let  $\vec{v}_i$  denote the velocity of node  $i$  and  $\bar{v}_{mm} = \|\vec{v}_i - \vec{v}_j\|$  be the mean relative speed between two nodes  $i$  and  $j$  when both are moving according to mobility model  $MM$ . Then we define the normalized relative speed  $\hat{v}_{mm}$  as  $\hat{v}_{mm} = \frac{\bar{v}_{mm}}{\bar{v}}$ .

The following formally defines hitting and meeting times.

**DEFINITION 3.1 (HITTING AND MEETING TIME).** *Let a node  $i$  move according to mobility process “MM”, and starting from its stationary distribution at time 0. Then,*

- i. If  $j$  is a static node with uniformly chosen  $X_j$ , then the expected hitting time under mobility model MM is  $ET_{mm} = \min_t \{t : \|X_i(t) - X_j\| \leq K\}$ .*
- ii. If  $j$  is a mobile node also starting from its stationary distribution, then the expected meeting time between the two nodes is  $EM_{mm} = \min_t \{t : \|X_i(t) - X_j(t)\| \leq K\}$ .*

Table 1 summarizes our notation.

We first analyze the hitting and meeting time for the Random Direction model. Although the Random Waypoint model was proposed first, it was quickly recognized that it results in a non-uniform stationary node distribution. This not only complicates analysis, as we shall see later, but was also in discord with the common assumption of uniformity made in many studies. To overcome this, the Random Direction model was proposed, which induces a uniform stationary node distribution [5], and makes it less difficult to analyze.

### 3.1 Random Direction Mobility Model

DEFINITION 3.2 (RANDOM DIRECTION). *In the Random Direction (RD) model each node moves as follows:*

1. choose a direction  $\theta$  uniformly in  $[0, 2\pi)$ ;
2. choose a speed according to assumption (d);
3. choose a duration  $T$  of movement from an exponential distribution with average  $\frac{\bar{L}}{\bar{v}}$ ;
4. move towards  $\theta$  with the chosen speed for  $T$  time units; If the network boundary is reached either reflect back or re-enter from the opposite side of the network (torus);
5. after  $T$  time units pause according to assumption (e) and go to step 1;

The following two Theorems calculate the expected hitting and meeting times for the Random Direction model. Our methodology is based on calculating the expected number of epochs until a static or mobile destination, respectively, is encountered.

THEOREM 3.1. *The expected time  $ET_{rd}$  for the Random Direction model is given by:*

$$ET_{rd} = \left( \frac{N}{2K\bar{L}} \right) \left( \frac{\bar{L}}{\bar{v}} + \bar{T}_{stop} \right). \quad (1)$$

PROOF. Let a node  $A$  perform RD movement, starting from its stationary distribution.  $A$ 's movement consists of a sequence of randomly and independently chosen epochs. Let further a second node  $B$  be static with uniformly chosen position, and let us calculate the probability that node  $A$  encounters node  $B$  during a given epoch  $i$  of length  $L_i$ . This epoch will "cover" an area of size  $2KL_i$ . If  $B$  lies anywhere within this area, then  $A$  "hits"  $B$  during this specific epoch. Furthermore, it is easy to see by the definition of the RD model, that the specific area of the network an epoch will cover is uniformly distributed around the whole network<sup>2</sup>. Hence, the probability  $p_i$  of an epoch of length  $L_i$  hitting  $B$  is equal to  $p_i = \frac{2KL_i}{N}$ .

Let us now denote as  $N_{hit}$  the number of epochs until  $A$  hits  $B$ , and  $P(N_{hit} > n)$  the probability that  $B$  has not been encountered after  $n$  epochs. Let further  $L_1, L_2, \dots, L_n$  denote the random sequence of lengths of these first  $n$  epochs, with joint probability density function equal to  $f_L(l_1, l_2, \dots, l_n)$ . Then:

$$P(N_{hit} > n) = \int \dots \int (1-p_1) \dots (1-p_n) f_L(l_1, \dots, l_n) dl_1 \dots dl_n.$$

The lengths  $L_i$  of different epochs are independent and distributed exponentially with average  $\bar{L}$ . Although consecutive epochs are not independent (the end of one epoch is the beginning of the next one), the random process describing the lengths and end-points of the sequence of epochs drawn is ergodic [6]. Thus, we can use the statistics of a single epoch to describe the whole process, as if the epochs were drawn independently (the argument is

<sup>2</sup>This is where the uniform node distribution in the RD model arises from.

similar to the one made for RWP in [6]). In other words,  $f_L(l_1, l_2, \dots, l_n) = f_L(l_1)f_L(l_2) \dots f_L(l_n)$ , and

$$P(N_{hit} > n) = \left( \int \left( 1 - \frac{2Kl}{N} \right) f_L(l) dl \right)^n = \left( 1 - \frac{2K\bar{L}}{N} \right)^n.$$

Consequently, the number of epochs needed till  $A$  hits  $B$  is geometrically distributed with average  $\frac{N}{2K\bar{L}}$ . Finally, the expected duration of each epoch is equal  $\bar{T} + \bar{T}_{stop}$  (see assumptions (e),(f)), where  $\bar{T} = \frac{\bar{L}}{\bar{v}}$  in the case of Random Direction.  $\square$

THEOREM 3.2. *The expected meeting time  $EM_{rd}$  for the Random Direction model is given by*

$$EM_{rd} = \frac{ET_{rd}}{p_m \hat{v}_{rd} + 2(1-p_m)}, \quad (2)$$

where  $\hat{v}_{rd} \approx 1.27$  is the normalized relative speed for RD, and  $p_m = \frac{\bar{T}}{\bar{T} + \bar{T}_{stop}}$  is the probability that a node is moving at any time.

PROOF. Let us first assume again that only one of the two nodes, let  $A$ , performs RD movement, while the second one, let  $B$ , is static. We will re-calculate the expected hitting time of Theorem 3.1 in a slightly different manner. Let's assume that node  $A$  performs RD movement *in discrete steps* of unit size, and let  $p_m$  denote the probability that  $A$  is moving at any of these steps. Then, with probability  $p_m$  any given step covers on average a new area of size  $2K\bar{v}$ , and with probability  $1-p_m$  it stands still and covers no new area. Thus, on average, each node step covers an area of  $p_m 2K\bar{v}$ , and the expected number of steps until the destination is found is equal to

$$ET'_{rd} = \frac{N}{p_m 2K\bar{v}}.$$

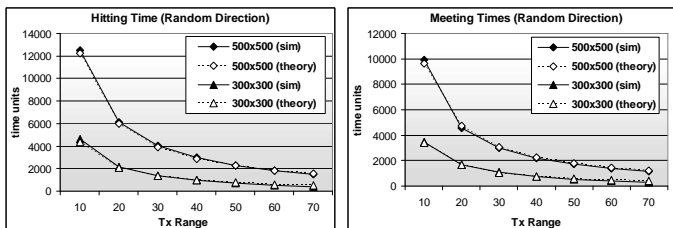
Note that this method of calculating the hitting time is equivalent to that of Theorem 3.1, i.e.  $ET'_{rd} = ET_{rd}$ <sup>3</sup>.

Now, to calculate the meeting time, we need to take into account that both  $A$  and  $B$  move *concurrently*. It is known that, for *generic* random walks on graphs the meeting time between two walks is  $\frac{1}{2}$  the respective hitting time of a single walk on the same graph [2]. This holds, because the movements at consecutive steps are independent of each other. However, in the RD model a node keeps moving in the same direction for the duration of an epoch, so the denominator is expected to be smaller than 2. We thus need to calculate the expected relative speed  $\|\vec{v}_A - \vec{v}_B\|$  between  $A$  and  $B$ . Due to the uniform choice of direction at every epoch, and the toroidal structure of the network, we can assume without loss of generality that the direction of  $\vec{v}_A$  is fixed. Then,

$$\|\vec{v}_A - \vec{v}_B\| = \frac{\bar{v}}{2\pi} \int_0^{2\pi} \sqrt{(1 + \cos \theta)^2 + \sin^2(\theta)} d\theta,$$

which is equal to  $1.27\bar{v}$ , where  $\hat{v}_{rd} = \frac{\|\vec{v}_A - \vec{v}_B\|}{\bar{v}} = 1.27$  is the normalized relative speed for RD.

<sup>3</sup>One can see this by replacing  $p_m$  with its value  $\frac{\bar{T}}{\bar{T} + \bar{T}_{stop}}$ , which then gives the expected hitting time in the familiar form of  $\frac{N}{2K\bar{L}} \left( \frac{\bar{L}}{\bar{v}} + \bar{T}_{stop} \right)$ .



**Figure 1: Comparison of theoretical and simulation results under the Random Direction model (0 pause time).**

$\hat{v}_{rd}\bar{v} = 1.27\bar{v}$  is the relative speed between the nodes when *both* nodes are moving, which occurs with probability  $p_m^2$ . However, with probability  $2p_m(1 - p_m)$  only one of the nodes moves with relative speed  $\bar{v}$ , and with probability  $(1 - p_m)^2$  none of the nodes is moving. Consequently, the expected number of steps until the two walks meet equals

$$\begin{aligned} EM_{rd} &= \frac{N}{2K(p_m^2\hat{v}_{rd}\bar{v} + 2p_m(1 - p_m)\bar{v})} \\ &= \frac{ET'_{rd}}{p_m\hat{v}_{rd} + 2(1 - p_m)}. \end{aligned}$$

□

Figure 1 compares analytical and simulation results for the expected hitting and meeting times, under the Random Direction model. (Note that in this and all other plots throughout we normalize average speed to  $\bar{v} = 1$ .)

### 3.2 Random Waypoint Mobility Model

In the original Random Waypoint mobility model (RWP) [9] a node chooses a point in the network uniformly and moves toward it with some speed. When it reaches that point it pauses for a random amount of time, chooses another point in the network uniformly, and so on and so forth:

**DEFINITION 3.3 (RANDOM WAYPOINT).** *A node moving according to the RWP model performs the following steps: (i) choose a point (“waypoint”)  $X$  in the network uniformly, (ii) move toward  $X$  with random speed chosen according to assumption (d), and (iii) when at  $X$ , pause a random amount of time according to assumption (e), and go to step 1.*

The stochastic properties of RWP have been extensively studied, as we mentioned earlier. In the following Lemma, we summarize some of these properties, derived in [6], which we use in our analysis.

**LEMMA 3.1.** *Let a node move according to RWP, and let  $f(x, y)$  denote the probability density function that a node is found at position  $(x, y)$  in the network. Then:*

- The expected epoch length  $\bar{L}$  can be derived in closed form (see [6]); for example, in a  $\sqrt{N} \times \sqrt{N}$  square,  $\bar{L} = 0.5214\sqrt{N}$ ;
- The stationary pdf  $f(x, y)$  on a  $\sqrt{N} \times \sqrt{N}$  square area is non-uniform, and can be approximated by

$$f(x, y) \approx \frac{36}{N^3} \left(x^2 - \frac{N}{4}\right) \left(y^2 - \frac{N}{4}\right). \quad (3)$$

A better, yet more convoluted approximation can be found in [7];

- The expected direction chosen at the beginning of an epoch is not uniform, but has a strong bias towards the center; Specifically, if  $\theta$  is the angle measured from the direction pointing to the center of the network ( $\theta = 0$ ), then the pdf  $f_{\Theta}(\theta)$  is given by

$$\begin{aligned} f_{\Theta}(\theta) &= \frac{1}{4\pi|\sin^3\theta|} [|\sin\theta|(-2\cos^4\theta - 2\cos^3\theta|\cos\theta| \\ &\quad + \cos^2\theta + \cos\theta|\cos\theta| + 1) + \arcsin(|\sin\theta|)\cos\theta]. \end{aligned}$$

An interesting property of the RWP model is that nodes tend to move and slowly concentrate towards the center. This introduces complications in the analysis, since the common assumption of uniform node distribution does not hold. For this reason, hitting and meeting times under RWP mobility have not been widely studied. To the best of our knowledge, the only related effort in that direction is that of [32]. There, the authors come up with upper and lower bounds for the hitting time, and then derive an asymptotic expression based on these bounds. Here, we derive an accurate closed form expression.

**THEOREM 3.3.** *Let  $f(x, y)$  denote the stationary probability density function of the position  $(x, y)$  of a node that performs RWP movement. Then, the expected hitting time  $ET_{rwp}$  under RWP mobility is given by:*

$$ET_{rwp} = \left( \frac{\iint_U \frac{1}{f(x, y)} dx dy}{2KN\bar{L}} \right) (\bar{T} + \bar{T}_{stop}). \quad (4)$$

PROOF. See Appendix. □

As can be seen by the above Theorem, calculating the hitting time under RWP movement introduces an integral involving the stationary pdf for RWP, whose exact value is not known [6]. Furthermore, the solution of the integral raises convergence issues, due to  $\frac{1}{f(x, y)} \rightarrow \infty$  at the edges of the network, and may not be calculable in closed form. The following Corollary of Theorem 3.3 uses the approximate expression of Eq.(3) for  $f(x, y)$  and takes care of the edge phenomena to provide an approximate yet very accurate closed form expression for the hitting time.

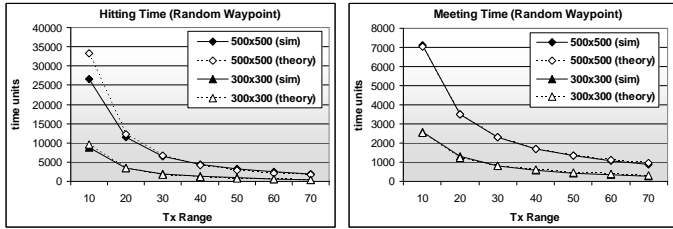
**COROLLARY 3.1.** *The expected hitting time under the Random Waypoint model can be approximated by the following expression:*

$$ET_{rwp} \approx \left( \frac{g(N, K)}{2KN\bar{L}} \right) (\bar{T} + \bar{T}_{stop}), \quad (5)$$

$$\text{where } g(N, K) = \frac{(N^{\frac{3}{2}} + 2(K - \sqrt{N})N \tanh^{-1}(1 - \frac{2K}{\sqrt{N}}))^2}{9(K - \sqrt{N})^2}.$$

PROOF. See Appendix. □

Despite the inherent difficulty in calculating the hitting time for this model, things become simpler when the destination performs RWP movement, as well. The position of  $B$  is not anymore uniform at any time, but is rather distributed according to  $f(x, y)$ . This, as we shall see, makes the meeting time expression independent of  $f(x, y)$ . In the following Theorem we derive the expected meeting time between two mobile nodes under RWP movement.



**Figure 2: Comparison of theoretical and simulation results under the Random Waypoint model.**

**THEOREM 3.4.** *The expected meeting time  $EM_{rwp}$  under Random Waypoint mobility is given by:*

$$EM_{rwp} = \frac{1}{p_m \hat{v}_{rwp} + 2(1 - p_m)} \frac{N}{2K\bar{L}} (\bar{T} + \bar{T}_{stop}), \quad (6)$$

where  $\hat{v}_{rwp} \approx 1.75$  is the normalized relative speed for RWP, and  $p_m = \frac{\bar{T}}{\bar{T} + \bar{T}_{stop}}$  is the probability that a node is moving at any time.

PROOF. See Appendix.  $\square$

In Figure 2 we evaluate our analytical results regarding the expected hitting and meeting time for the Random Waypoint model against simulation results. As can be seen there both expressions have a very good match with simulation results. The minor discrepancy in hitting time for very small transmission ranges is due to an approximation we make in the proof to account for edge phenomena (see proof of Corollary 3.1 in the Appendix).

#### 4. COMMUNITY-BASED MOBILITY MODEL

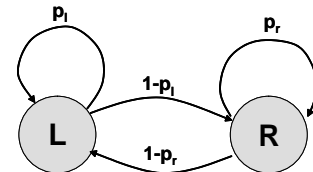
In order to have a mobility model that better resembles real node movement, yet is still analytically tractable, we propose the ‘‘Community-based Mobility Model’’. This model consist of two states/phases, namely ‘‘local’’ state and ‘‘roaming’’ state, between which it alternates:

**DEFINITION 4.1 (COMMUNITY-BASED MODEL).** *Nodes move inside the network as follows:*

- each node  $i$  has a local community  $C_i$  of size  $\|C_i\| = c^2N, c \in (0, 1]$ ; a node’s movement consists of a sequence of local and roaming epochs.
- a **local epoch** is a Random Direction movement restricted inside area  $C_i$  with average epoch length  $\bar{L}_c$  equal to the expected distance between two points uniformly chosen in  $C_i$ .
- a **roaming epoch** is a Random Direction movement inside the entire network with expected length  $\bar{L}$ .
- (local state L) if the previous epoch of node  $i$  was a local one, the next epoch is a local one with probability  $p_l^{(i)}$ , or a roaming epoch with probability  $1 - p_l^{(i)}$ .
- (roaming state R) if the previous epoch of node  $i$  was a roaming one, the next epoch is a roaming one with probability  $p_r^{(i)}$ , or a local one with probability  $1 - p_r^{(i)}$ .

**Table 2: Additional Notation for Section 4**

$C_i$	community of node $i$ : $\ C_i\  = c^2N, c \in (0, 1]$
$p_l^{(i)}$	probability that next epoch is local, given that previous epoch was local
$p_r^{(i)}$	probability that next epoch is roaming, given that previous epoch was roaming
$\pi_l^{(i)}$	probability that a given epoch is a local one
$\pi_r^{(i)}$	probability that a given epoch is a roaming one
$\bar{L}_c$	expected length of local epoch
$\bar{T}_{stop}^l$	expected pause time for a local epoch
$\bar{T}_l$	expected local epoch duration ( $\bar{L}_c/\bar{v} + \bar{T}_{stop}^l$ )
$\bar{T}_r$	expected roaming epoch duration ( $\bar{L}/\bar{v} + \bar{T}_{stop}^l$ )



**Figure 3: Community-based Mobility Model**

The Community-based mobility model can be represented by the simple two-state Markov Chain depicted in Figure 3. (Note that each node could also perform Random Waypoint movement during each of these states, instead of Random Direction.) In [24] a mobility model where each node has its own community to which it may move preferentially is also proposed. However, here each node may also have different  $p_r^{(i)}$  and  $p_l^{(i)}$  parameters capturing different mobility characteristics of different nodes. Furthermore, unlike [24], different nodes may have communities of very different sizes. These together allow for a much larger range of node heterogeneity to be captured.

Let us denote as  $\pi_l^{(i)}$  and  $\pi_r^{(i)}$  the probability that a given epoch of node  $i$  is a local or a roaming one, respectively, then from elementary Markov chain theory we get that

$$\pi_l^{(i)} = \frac{1 - p_r^{(i)}}{2 - p_l^{(i)} - p_r^{(i)}} \text{ and } \pi_r^{(i)} = \frac{1 - p_l^{(i)}}{2 - p_l^{(i)} - p_r^{(i)}}.$$

Table 2 summarizes some additional notation related to the community model.

##### 4.1 Hitting Times under Homogeneous Mobility

We will first calculate the expected hitting and meeting times when each node  $i$  has its own community  $C_i$  yet all nodes have the same mobility characteristics, that is,  $p_l^{(i)} = p_l$  and  $p_r^{(i)} = p_r, \forall i$ . We deal with heterogeneous mobility in the next section.

Let’s assume that a node  $A$  with community  $C_A$  moves according to the Community-based model, until it encounters a node  $B$  that is static with uniformly chosen position. If  $B$ ’s position is outside  $C_A$ , then  $A$  can only encounter  $B$  during a roaming epoch. Otherwise, if  $B$  lies inside  $C_A$ ,  $A$  is expected to encounter  $B$  much faster, since it tends to move preferentially inside  $C_A$ . The following two Lemmas calculate the expected hitting time for each of these two subcases.

LEMMA 4.1. *Let a node A move according to the Community based Random Direction model. Let further B denote a second, static node, whose position is uniformly distributed outside A's community  $C_A$ . Then, the expected hitting time  $ET_{comm}^{(out)}$  until A encounters B, is given by:*

$$ET_{comm}^{(out)} = ET_{rd} + \frac{1 - p_r}{1 - p_l} \frac{N}{2KL} \bar{T}_l. \quad (7)$$

PROOF. Let  $N_l$  and  $N_r$  denote the number of times A visits the local state (L) and roaming state (R), respectively, before it finds B. Furthermore, let  $N_{hit} = N_l + N_r$  denote the total number of epochs of any kind. Then, according to the law of large numbers, when  $N_{hit} \rightarrow \infty$ ,  $N_l \rightarrow \pi_l N_{hit}$  and  $N_r \rightarrow \pi_r N_{hit}$ .

Since B does not lie inside A's community, B can only be encountered while A is in the roaming state<sup>4</sup>. The expected number of roaming epochs needed until such a destination is met was found in Theorem 3.1, to be equal to  $\frac{2K\bar{L}}{N}$ . This implies that A visits state R  $E[N_r] = \frac{2K\bar{L}}{N}$  number of times, before it meets B. The sum of the duration of these epochs is equal to  $ET_{rd}$ . Additionally, according to the previous argument based on the law of large numbers, A also visits state L on average

$$E[N_l] = \frac{\pi_l}{\pi_r} E[N_r] = \frac{1 - p_r}{1 - p_l} E[N_r]$$

times, before it meets B (given that A starts from its stationary distribution). The average time spent at state L every time it is visited is equal to  $\frac{\bar{L}_c}{\bar{v}} + \bar{T}_{stop}^l$ . Putting everything together gives us Eq.(7).  $\square$

LEMMA 4.2. *Let a node A move according to the Community based Random Direction model. Let further B denote a second, static node, whose position is uniformly distributed inside A's community  $C_A$ . Then, the expected hitting time  $ET_{comm}^{(in)}$  until A encounters B, is given by:*

$$ET_{comm}^{(in)} \approx \frac{1}{1 - [(1 - p_{hit}^l)^{\pi_l} (1 - p_{hit}^r)^{\pi_r}]} (\pi_l \bar{T}_l + \pi_r \bar{T}_r), \quad (8)$$

where  $p_{hit}^r = \frac{2K\bar{L}}{N}$  and  $p_{hit}^l = \frac{p_{hit}^r}{c}$ .

PROOF. Let us count the number of steps in the Markov chain corresponding to the community model until B is found. Let further  $N_l$  and  $N_r$  denote again the number of local and roaming epochs elapsed, respectively, before B is encountered, and let  $N_{hit} = N_l + N_r$  denote the total number of epochs. Then, the probability  $P(N_l, N_r)$  that at least  $N_l$  local and  $N_r$  roaming epochs elapse before B is found is equal to  $P(N_l, N_r) = (1 - p_{hit}^l)^{N_l} (1 - p_{hit}^r)^{N_r}$ . Furthermore, according to the law of large numbers:

$$\lim_{N_{hit} \rightarrow \infty} P(N_l, N_r) = P(N_{hit} > n) = (1 - p_{hit}^l)^{\pi_l n} (1 - p_{hit}^r)^{\pi_r n}.$$

This implies that the probability distribution of the total number of epochs  $N_{hit}$  (local or roaming) has a geometric tail with parameter

$$p_{hit} = 1 - [(1 - p_{hit}^l)^{\pi_l} (1 - p_{hit}^r)^{\pi_r}].$$

<sup>4</sup>we assume that the transmission range  $K$  of nodes is much smaller than the total network area  $N$ , and thus the probability that B is near the edge of  $C_A$  and thus can be encountered even while A is inside its community goes to 0 as  $N \rightarrow \infty$ .

Consequently, when the average number of epochs necessary to find B is not too small, we can approximate the pdf of the total epochs with a geometric distribution with the above parameter  $p_{hit}$ . For this to occur we require that the transmission range is much smaller than the network dimensions, which is the case indeed in most situations of interest (i.e. when mobility is required to deliver a message). In this case, the expected number of epochs until B is encountered  $EN_{hit}$  is equal to  $\frac{1}{p_{hit}}$ . Finally, each of these epochs is a local one with probability  $\pi_l$  or a roaming one with probability  $\pi_r$ , and with duration  $\bar{T}_l$  and  $\bar{T}_r$ , respectively.  $\square$

We can now go ahead and calculate the hitting time for the case where the destination's position is uniformly chosen over the entire network area.

THEOREM 4.1. *The expected hitting time  $ET_{comm}$  under the Community-based Mobility Model is given by:*

$$ET_{comm} = (1 - c^2) ET_{comm}^{(out)} + c^2 ET_{comm}^{(in)}. \quad (9)$$

PROOF. With probability  $\frac{\|U\| - c^2 \|U\|}{\|U\|} = 1 - c^2$  B's position is outside A's community  $C_A$ . In that case, B can only be encountered during a roaming phase, and the expected time until this occurs is given is  $ET_{comm}^{(out)}$  (Lemma 4.1). Similarly, with probability  $c^2$  B lies inside  $C_A$ , in which case the expected hitting time is given by Lemma 4.2.  $\square$

Finally, Theorem 4.2 gives the expected meeting time when both nodes are moving.

THEOREM 4.2. *The expected meeting time  $EM_{comm}$  under the Community-based Random Direction model is given by:*

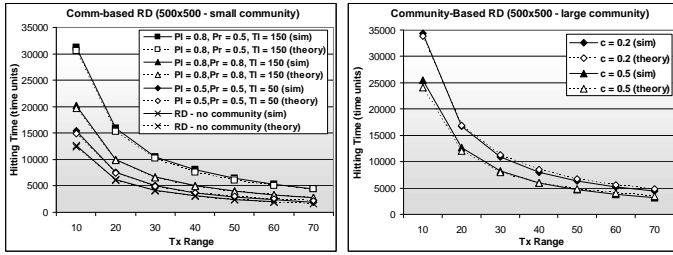
$$EM_{comm} = \frac{ET_{comm}}{p_m^c \hat{v}_{rd} + 2(1 - p_m^c)}, \quad (10)$$

where  $p_m^c = \frac{(1 - p_r) \bar{L}_c / \bar{v} + (1 - p_l) \bar{L} / \bar{v}}{(1 - p_r) \bar{T}_l + (1 - p_l) \bar{T}_r}$  is the probability that a node is moving at any time.

PROOF. See Appendix.  $\square$

So far we have assumed that the community of each node could be quite large, covering a considerable chunk of the network area. This might be the case for example when the community is a department building in a small college campus network, which implies that two nodes have a good chance of sharing the same community. However, in many real-life situations, each node tends to move most of the time in a very small area that's different for each node (e.g. at home), and that could be entirely covered by the node's antenna, while the network might be much larger (e.g. a city-wide wireless Metropolitan Area Network). In such situations, the probability that two nodes share the same community can be neglected. The following Corollary of Theorem 4.2 calculates the meeting time for the special case of small communities.

COROLLARY 4.1 (SMALL COMMUNITY). *When the community size of nodes is much smaller than the network area*



**Figure 4: Hitting Times under the Community-based RD model for small (left) and large (right) communities.**

( $\|C_i\| \ll N$ ), the expected meeting time  $EM_{comm}^{(small)}$  under the Community-based Random Direction model is given by:

$$EM_{comm}^{(small)} = \frac{ET_{rd} + \frac{1-p_r}{1-p_l} \frac{N}{2KL} \bar{T}_{stop}^l}{p_m^c \hat{v}_{rd} + 2(1-p_m^c)}, \quad (11)$$

$$\text{where } p_m^c = \frac{(1-p_l)\bar{L}/\bar{v}}{(1-p_r)\bar{T}_{stop}^l + (1-p_l)\bar{T}_r}.$$

PROOF. Eq.(11) follows easily from Eq.(10),(9), and (7) by replacing  $c \approx 0$  and  $\bar{T}_l \approx \bar{T}_{stop}^l$ .  $\square$

Figure 4 compares analytical and simulation results for the expected hitting time under the Community-based Random Direction model, for small and large communities (for the large community case all pause times are zero and  $P_l = 0.9, P_r = 0.5$ ). As can be seen there theory matches simulations quite closely. We have also observed similar accuracy for the respective meeting time results.

## 4.2 Hitting Time for Heterogeneous Nodes

In the following theorem we calculate the expected meeting time in a heterogeneous community model, where each node (i) has its own community, and (ii) has its own mobility parameters. This is a model where nodes do not move in an identical manner.

**THEOREM 4.3.** *Let two nodes  $A$  and  $B$  move according to the Community-based Random Direction, each with its own community  $C_i$  and model parameters  $(p_l^{(i)}, p_r^{(i)})$ ,  $i \in \{A, B\}$ . Let further  $ET_{comm}(i \rightarrow j)$  denote the expected hitting time under the Community-based model for the case where only node  $i$  is mobile and  $j$  is static. Then the expected meeting time  $EM_{comm}^{(h)}$  between  $A$  and  $B$  is given by*

$$EM_{comm}^{(h)} = \frac{ET_{comm}(A \rightarrow B)}{p_m^{(B)} \hat{v}_{rd} + (1-p_m^{(B)}) + \frac{p_m^{(B)}}{p_m^{(A)}}(1-p_m^{(A)})} \quad (12)$$

$$= \frac{ET_{comm}(B \rightarrow A)}{p_m^{(A)} \hat{v}_{rd} + (1-p_m^{(A)}) + \frac{p_m^{(A)}}{p_m^{(B)}}(1-p_m^{(B)})}, \quad (13)$$

$$p_m^i = \frac{(1-p_r^{(i)})\bar{L}_c/\bar{v} + (1-p_l^{(i)})\bar{L}/\bar{v}}{(1-p_r^{(i)})\bar{T}_l + (1-p_l^{(i)})\bar{T}_r}.$$

PROOF. See Appendix.  $\square$

It is important to note that Theorem 4.3 can be generalized to capture heterogeneity in any of the previous models discussed. For example, let two nodes  $A$  and  $B$  move according to any waypoint-based mobility model  $MM$ , but with different mobility characteristics (e.g. RWP movement with different pause time or different average speed), and let  $E_A T_B^{(mm)}$  and denote the respective hitting times from  $A$  to  $B$ , under the given mobility model. Then the expected meeting time  $EM^{(mm)}$  between the two non-identically moving nodes is given by

$$EM^{(mm)} = \frac{E_A T_B^{(MM)}}{p_m^{(B)} \hat{v}_{mm} + (1-p_m^{(B)}) + \frac{p_m^{(B)}}{p_m^{(A)}}(1-p_m^{(A)})},$$

where  $p_m^A$  and  $p_m^B$  denote the probabilities that nodes  $A$  and  $B$ , respectively, are moving at any given time instant. We have validated the above analytical expressions against simulations, as well, and have observed similar good accuracy.

## 5. DELAY OF MOBILITY-ASSISTED ROUTING

In this section, we will look into how our results regarding the meeting times can be integrated with a general theoretical framework that can be used to analyze the performance of mobility-assisted routing. Our goal is to demonstrate that our meeting time results can be easily plugged into generic equations regarding the delay of different routing schemes, and derive performance results for a specific mobility model without resorting to simulations or curve fitting.

### 5.1 Upper and Lower Bounds on Delay

The meeting time for a given mobility model corresponds to the expected delay of the simplest imaginable mobility-assisted routing scheme, namely Direct Transmission. In Direct Transmission, the source of a message holds on to it until it comes within range of the destination itself. This scheme has the largest expected delay among all possible encounter-based schemes (that are non-contrived and non-adversarial). Consequently, the meeting times derived in the previous section for a given mobility model constitute an upper bound on the delay of any scheme under this model.

Additionally, the algorithm that minimizes the expected delivery delay is an ‘oracle-based’ algorithm. It is aware of all future movement of nodes and computes the *optimal set of forwarding decisions* (i.e. time and next hop), which delivers a message to its destination in the minimum amount of time. This algorithm cannot be implemented in reality. However, we can use its delay as a lower bound on the performance of all routing strategies. Note also that, under the assumption of no contention (i.e. infinite buffer capacity and bandwidth), an algorithm called *epidemic routing* will find the same paths as the oracle-based scheme, and thus achieve this minimum expected delay. In epidemic routing, whenever a node encounters another one, they exchange all messages that they don’t have in common [38]. This algorithm is essentially flooding in this context, and its properties have been widely studied [29, 31, 33, 35, 42]. The following Lemma bounds the expected delay of any mobility-assisted routing scheme, under a given mobility model (proofs can be found in any of the above papers).

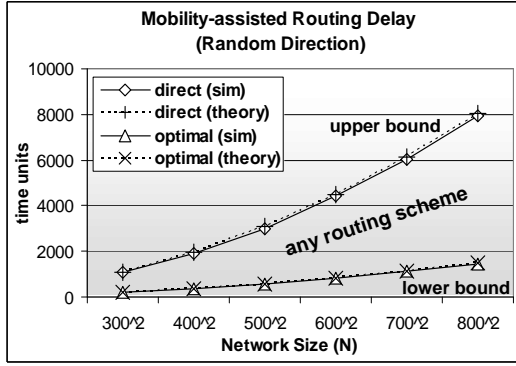


Figure 5: Upper and lower bounds on the delay of any mobility-assisted routing scheme under Random Direction mobility.

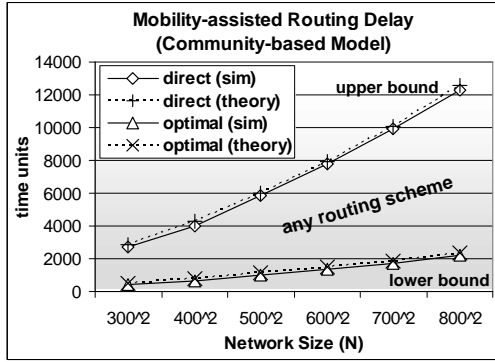


Figure 6: Upper and lower bounds on the delay of any mobility-assisted routing scheme under Community-Based mobility.

LEMMA 5.1. Let  $M$  nodes move according to a given mobility model with exponentially distributed meeting times. Then, the expected message delivery time of any routing algorithm  $ED$  is

$$\frac{H_{M-1}}{(M-1)} EM^{mm} \leq ED \leq EM^{mm} \quad (14)$$

where  $H_n$  is the  $n^{\text{th}}$  Harmonic Number, i.e.,  $H_n = \sum_{i=1}^n \frac{1}{i} = \Theta(\log n)$ .

Lemma 5.1 implies that we can replace the values we calculated for the meeting times under different mobility models in Eq.(14) and derive closed-form expressions for the best-case and worst-case delays of any mobility-assisted scheme, under the given mobility model<sup>5</sup>. To the best of our knowledge, this has only been performed for the case of Random Walk mobility [35]. For other mobility models (e.g. Random Waypoint, etc.) no such closed form expressions exist. Instead, simulations are run for a specific set of network parameters to estimate  $EM^{mm}$  directly or by fitting the theoretical curves to the simulation ones.

In Figure 5 we compare our analytical results, based on Lemma 5.1 and the expressions derived in Sections 3, 4,

<sup>5</sup>In the case of Community-based mobility with large communities Eq.(14) has to be slightly modified; However, the resulting expression is more convoluted without providing much additional insight, so we choose to omit it.

to simulation results, for the Random Direction model (0 pause times). Figure 6 does the same for the Community-based Random Direction model ( $p_l = 0.8$ ,  $p_{roam} = 0.5$ ,  $\bar{T}_{stop} = 0$ ,  $\bar{T}_{stop}^j = 150$ ) with small communities. As can be seen by both plots, our theoretical results for the optimal delay match very closely with simulations results. (We have observed similar good accuracy for the respective Random Waypoint results, so we do not include plots for it due to the limited space.) This implies not only that our meeting time expressions for different mobility models are accurate, but that *derivative delay expressions based on these meeting times, and pertaining to the delay of more complicated mobility-assisted routing schemes are also accurate.*

## 5.2 Delay of Other Routing Schemes

In this section, we're going to look into a couple of examples of mobility-assisted routing scheme, namely a single-copy (i.e. only one copy of each message is routed) and a multiple-copy one (i.e. more than one copies of the same message are routed in parallel). We first look into a multiple copy scheme, where the source of a message distributes only a fixed number of copies  $L$ , one to each of the first  $L$  distinct relays it encounters. Although a number of different names have been used for variants of this scheme, we'll refer to it here simply as the  $L$ -copy scheme. This scheme has been found to achieve delays comparable to the optimal scheme, without the overhead of transmitting a copy to every node in the network. Different approximations and bounds for its delay have been calculated (e.g. [31,33,34]). Lemma 5.2 gives a recursive method, first proposed in [34] for Random Walk mobility, and adapted here for any mobility model, to calculate the delay of the  $L$ -copy scheme.

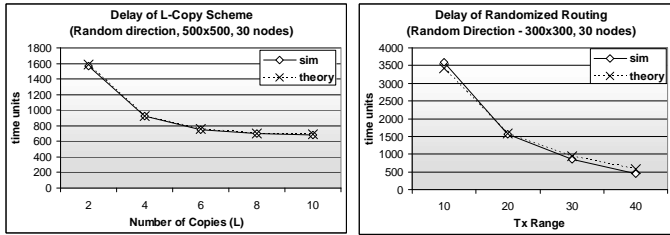
LEMMA 5.2. Let  $ED_L^{mm}$  denote the expected delay of the  $L$ -copy scheme, under mobility model "MM". Let further  $ED(i)$  denote the expected remaining delay for this scheme, after  $i$  message copies have been distributed. Then,  $ED_L^{mm}$  can be calculated by the following system of recursive equations, where  $ED_L^{mm} \equiv ED(1)$ :

$$\begin{aligned} ED(i) &= \frac{EM_{mm}}{M-1} + \frac{M-i}{M-1} ED(i+1), i \in [1, L-1], \\ ED(L) &= \frac{EM_{mm}}{L}. \end{aligned} \quad (15)$$

Next, we analyze the expected delay for a scheme that uses only a single copy per message.

DEFINITION 5.1 (RANDOMIZED ROUTING). In the randomized routing algorithm  $p$ , a node  $A$  hands over a message to another node  $B$  it encounters with probability  $p > 0$ .

Randomized routing could be thought of as Direct Transmission, with the difference that a message may occasionally jump to a new relay, whenever such an appropriate relay is encountered. This and other similar single-copy schemes [35] that occasionally handover the message to a new relay based on some criteria can be analyzed using the methodology of Theorem 3.2. Due to lack of space we only present results for the delay of Randomized Routing under RD mobility and we omit the proof. Similar results can be derived for the rest of the mobility models, as well.



**Figure 7: Simulation and analytical results for the expected delay of Randomized and L-copy routing schemes.**

LEMMA 5.3. Let  $M$  nodes perform Random Direction mobility, and let  $\bar{v}_{rd}$  denote the average relative speed between two nodes. Let further a node  $A$  have a message to deliver to another node  $B$ . Then, the expected time  $ED_{rand}$  until the message is delivered to  $B$  using Randomized routing with probability  $p$  is given by:

$$ED_{rand} = \frac{N}{2K [(1 - pp_{tx})\bar{v}_{rd} + pp_{tx}\frac{2}{3}K]}, \quad (16)$$

where  $p_{tx} = 1 - \left(1 - \frac{\pi K^2}{N}\right)^{M-1}$  is the probability that at least one node is within range of the current message relay at any time, and  $\bar{v}_{rd} = p_m^2 \hat{v}_{rd} \bar{v} + 2p_m(1 - p_m)\bar{v}$ .

Figure 7 compares analytical and simulation results for the expected delay of Randomized routing with  $p = 1$  (left plot) and the L-copy scheme (right plot), under Random Direction mobility with 0 pause times.

## 6. CONCLUSIONS

In this paper, we have analyzed the encounter statistics for some commonly used mobility models. We have derived accurate closed form solutions for all the respective hitting and meeting times between different nodes. Additionally, we have proposed a mobility model that is rich enough to capture real-world mobility characteristics more accurately than many existing models, and have calculated various hitting time results for it, as well. Finally, we have demonstrated how these results can be used in a more general framework to analyze the delay of different mobility-assisted routing schemes, that is, schemes that require the node to carry a message for (potentially long) periods of time. Such schemes have been recently recognized to be very helpful in improving the performance of regular wireless networks or to enable data delivery in networks that are disconnected for the majority of time. We believe that this work can help in better understanding the particular advantages and shortcomings of various approaches in different settings, and can facilitate the design of new, improved protocols.

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## APPENDIX

PROOF. (**Theorem 3.3**) Let a node  $A$  perform RWP movement, starting from its stationary distribution, and let a second node  $B$  be static with uniformly chosen position. Let us further look at a randomly chosen epoch with endpoints  $X_s$  and  $X_f$ , and let us denote as  $X_{s-f}^\perp$  the point of line  $(X_s, X_f)$  that's closest to  $B$ . If we also denote the epoch with these endpoints as  $(X_s, X_f)$  then  $B$  is hit by epoch  $(X_s, X_f)$  if and only if  $\|X_B - X_{s-f}^\perp\| \leq K$ . The probability of this event occurring depends on  $X_B = (x_B, y_B)$ , because the probability distribution of the position of  $B$  is not uniform. We denote this probability as  $P_{hit|B}$ .  $P_{hit|B}$  is higher when  $B$  is near the center and smaller when  $B$  is near the edges.

Let now  $S$  denote the set of all possible epochs, that is,  $S = \{(X_s, X_f) : X_s, X_f \in U\}$ . Let further  $S_{hit|B}$  denote the set of all epochs that "hit"  $X$ , that is,  $S_{hit|B} = \{(X_s, X_f) : \|X_B - X_{s-f}^\perp\| \leq K\}$ . The size of a given set of

epochs  $S_e$  is calculated by  $\|S_e\| = \iint_{S_e} 1dX_s dX_f$ . It is not difficult to see then, that  $P_{hit|B}$  is given by:

$$P_{hit|B} = \frac{\|S_{hit|B}\|}{\|S\|} = \frac{\|S_{hit|B}\|}{N^2}.$$

Using a similar argument as in the proof of Theorem 3.1, we can use the statistics of a single epoch to calculate the expected number of epochs until  $A$  comes within range of  $B$ . Thus, if  $N_{hit|B}$  is a random variable denoting the number of epochs until  $B$  is encountered, conditioned on  $B$ 's position, then  $N_{hit|B}$  is geometrically distributed with mean  $EN_{hit|B} = \frac{N^2}{\|S_{hit|B}\|}$ . Averaging over all possible locations  $X_B = (x_B, y_B)$  (using linearity of expectations) we calculate that the unconditional expected number of epochs  $EN_{hit}$  is

$$\begin{aligned} EN_{hit} &= \iint_U EN_{hit|B} P\{X_B = (x_B, y_B)\} dx_B dy_B \\ &= \iint_U \frac{N^2}{\|S_{hit|B}\|} \frac{1}{N} dx dy = \iint_U \frac{N}{\|S_{hit|B}\|} dx dy. \end{aligned}$$

Let us now define the set  $S_{hit|B}^*$ , which contains all *lines* in  $U$  with end-points drawn uniformly, whose  $X_B$  is a part. In other words  $S_{hit|B}^* = \{(X_s, X_f) : \|X_B - X_{s-f}^\perp\| = 0\}$ . It is not difficult to notice that  $\|S_{hit|B}\| \approx 2K\|S_{hit|B}^*\|$ . Furthermore,  $f(x, y) = \frac{\|S_{hit|B}^*\|}{N^2 L}$ . Consequently,  $\|S_{hit|B}\| \approx \frac{2KN^2 L}{f(x, y)}$ , and the expected number of epochs  $EN_e$  until  $A$  encounters a static node  $B$  is equal to

$$\frac{\iint_U \frac{1}{f(x, y)} dx dy}{2KNL}$$

Finally, the expected duration of each of these epochs is given in assumption (d).  $\square$

PROOF. (**Corollary 3.1**) As is shown in [6] Eq.(3) is quite a good approximation of the stationary pdf  $f(x, y)$  away from the network edges. However, we cannot simply replace this value in the hitting time equation and calculate its value, because the integral  $\iint_U \frac{1}{f(x, y)} dx dy$  would not converge. The reason for this is that  $f(x, y) \rightarrow 0$  at the network edges, implying that if the destination is very near the edge, it would take an infinite number of epochs to find it. However, this is not the case in reality, since nodes have a non-zero transmission range  $K$ . Hence, we can assume that the value of  $\frac{1}{f(x, y)}$  everywhere inside a strip of width  $K$  from the network edge is constant and equal to its value  $K$  distance far from the edge. Referring to the proof of Theorem 3.3 this simply implies that the probability of a given epoch hitting a point is constant inside that strip (this assumption becomes increasingly valid as  $K$  increases).

Based on this observation, we can now brake the integral as follows:

$$\iint_U \frac{1}{f(x, y)} dx dy = \frac{36}{N^3} (I_1^2 + 4I_1 I_2 + 4I_2^2),$$

where

$$I_1 = \int_{-\sqrt{N}/2+K}^{\sqrt{N}/2-K} (x^2 - N/4)^{-1} dx,$$

$$I_2 = \int_{\sqrt{N}/2-K}^{\sqrt{N}/2} \left[ \left( \sqrt{N}/2 - K \right)^2 - N/4 \right]^{-1} dx.$$

Using some calculus we can easily calculate the values of  $I_1$  and  $I_2$  as

$$I_1 = \frac{(4 \tanh^{-1}(1 - \frac{2K}{\sqrt{N}}))^2}{\sqrt{N}}, \text{ and } I_2 = \frac{1}{K - \sqrt{N}}.$$

Replacing these values in the above integral equation and performing some calculations gives us  $g(N, K)$ .  $\square$

**PROOF. (Theorem 3.4)** When the destination, let  $B$ , is also moving according to the Random Waypoint model, the position  $X_B$  of the destination at any time is distributed according to  $f(x, y)$ , instead of uniformly ( $f(x, y)$  again denotes the stationary node distribution for RWP). Let us then first calculate the expected hitting time until a static node, distributed according to  $f(x, y)$  is encountered by  $A$ . Let's also assume initially that the pause time at every epoch is 0 (i.e.  $\bar{T}_{stop} = 0$ ). Finally, let  $N_{hit|B}$  be again the random variable denoting the number of epochs until  $B$  is encountered, conditioned on  $B$ 's position, and  $N_{hit}$  be the respective unconditional variable. We can now change the steps of the proof of Theorem 3.3 accordingly to calculate the expected number of epochs  $EN_{hit}$  until  $B$  is found as follows:

$$\begin{aligned} EN_{hit} &= \iint_U N_{hit|B} P\{X_B = (x, y)\} dx dy \\ &= \iint_U \frac{N^2}{\|S_{hit|B}\|} f(x, y) dx dy \\ &= \iint_U \frac{1}{2K\bar{L}f(x, y)} f(x, y) dx dy \\ &= \frac{N}{2K\bar{L}}, \end{aligned}$$

since  $S_{hit|B} = \frac{2K\bar{L}}{N^2} f(x, y)$ . ( $EN_{hit}$  is smaller here than when  $B$ 's position was uniformly distributed.) Taking into account that the expected epoch duration is given by  $\frac{\bar{L}}{u}$  we get that the expected hitting time until a static node, distributed according to stationary RWP pdf, is encountered is  $\frac{N}{2K\bar{v}}$ .

Now, if  $A$  also pauses at the end of every epoch (i.e.  $T_{stop} > 0$ ), we calculate the expected hitting time using the alternative method of Theorem 3.2, and find it equal to  $ET'_{rwp} = \frac{N}{2Kp_m\bar{v}}$ . The rest of the proof is the same as that of Theorem 3.2, only replacing the normalized relative speed  $\hat{v}_{rwp}$  for  $\hat{v}_{rd}$ . However, note that, to calculate  $\hat{v}_{rwp}$ , we need to integrate over the angle distribution for the velocities of both  $A$  ( $\vec{v}_A$ ) and  $B$  ( $\vec{v}_B$ ) from Lemma 3.1. Unlike the Random Direction case, this needs to be done numerically and gives that  $\hat{v}_{rwp} \approx 1.754$ .  $\square$

**PROOF. (Theorem 4.2)** The proof is similar to that of Theorem 3.2. We only need to recalculate the respective

probability of a node moving for the community model,  $p_m^c$ , at any time. Each local epoch last on average  $\bar{T}_l = \frac{\bar{L}_c}{\bar{v}} + \bar{T}_{stop}^l$  time units, and each roaming epoch  $\bar{T}_r = \frac{\bar{L}}{\bar{v}} + \bar{T}_{stop}$  time units. Furthermore, it is easy to see (as in all previous cases) that the community-based mobility process is also ergodic. Consequently, the probability  $p_m^c$  is equal to the percentage of time a node is in a moving state in an infinite evolution of the mobility process. Let then  $N_r$  and  $N_l$  denote the total number of roaming and local epochs having occurred in the above evolution, and  $N_{hit} = N_r + N_l$  denote the total number of epochs till the destination is found. In this case

$$p_m^c = \lim_{N_{hit} \rightarrow \infty} \frac{N_l \frac{\bar{L}_c}{\bar{v}} + N_r \frac{\bar{L}}{\bar{v}}}{N_l \bar{T}_l + N_r \bar{T}_r}.$$

We know also that  $N_r \rightarrow \pi_r N_{hit}$  and  $N_l \rightarrow \pi_l N_{hit}$  as  $N_{hit} \rightarrow \infty$  (law of large numbers). Hence,

$$p_m^c = \frac{\pi_l \frac{\bar{L}_c}{\bar{v}} + \pi_r \frac{\bar{L}}{\bar{v}}}{\pi_l \bar{T}_l + \pi_r \bar{T}_r} = \frac{(1 - p_r) \frac{\bar{L}_c}{\bar{v}} + (1 - p_l) \frac{\bar{L}}{\bar{v}}}{(1 - p_r) \bar{T}_l + (1 - p_l) \bar{T}_r}$$

$\square$

**PROOF. (Theorem 4.3)** Let us denote as  $EN^{(0)}$  the expected number of epochs till hitting time, when node  $A$  moves constantly (i.e. pause time in any phase is 0). Using a similar argument as in Lemma 3.4, when node  $A$  pauses also between epochs, then the resulting hitting time becomes

$$ET = \frac{EN^{(0)}}{p_m^{(A)} \bar{v}},$$

since  $p_m^{(A)} \bar{v}$  is the expected area covered within a single time unit. When node  $B$  is also mobile we need to take into account the fact that either both nodes are mobile (probability  $p_m^A p_m^B$ ) and move with  $\hat{v}_{rd} \bar{v}$  relative speed (see Theorem 3.2), or only one of the two nodes is moving (probability  $p_m^A (1 - p_m^B) + p_m^B (1 - p_m^A)$ ) with relative speed  $\bar{v}$ , or both nodes stand (probability  $(1 - p_m^A)(1 - p_m^B)$ ) with relative speed 0. Consequently, the expected meeting time  $EM$  will be given by

$$\begin{aligned} EM &= \frac{EN^{(0)}}{p_m^A p_m^B \hat{v}_{rd} \bar{v} + [p_m^A (1 - p_m^B) + p_m^B (1 - p_m^A)] \bar{v}} \\ &= \frac{ET}{p_m^{(B)} \hat{v}_{rd} + (1 - p_m^{(B)}) + \frac{p_m^{(B)}}{p_m^{(A)}} (1 - p_m^{(A)})}. \end{aligned}$$

The procedure is entirely symmetrical, so we could use the hitting time of  $B$  to  $A$ , instead, replacing the appropriate quantities in the above equations.  $\square$