Dynamic Factor Models with Time-Varying Parameters

Measuring Changes in International Business Cycles *

Marco Del Negro
Federal Reserve Bank of New York

Christopher Otrok
University of Virginia

First Version: February 2003; This Version: July 2008

Abstract

We develop a dynamic factor model with time-varying factor loadings and stochastic volatility in both the latent factors and idiosyncratic components. We apply this novel measurement tool to study the evolution of international business cycles during the post-Bretton Woods period in terms of changes in both volatility and synchronization, using a panel of output growth rates for 19 countries. In contrast with the previous literature, our approach does not rely on rolling window/subsample estimates, or univariate analysis, but explicitly models time-variation in the coefficients and, at the same time, considers all countries jointly.

JEL CLASSIFICATION: C11, C32, F02

KEY WORDS: Bayesian Factor Models; Time-Varying Parameters; Great Moderation; International Business Cycles.

---

*Marco Del Negro: FRBNY, 33 Liberty Street, NY NY 10045. marco.delnegro@ny.frb.org; Christopher Otrok, Department of Economics, University of Virginia, PO Box 400182, Charlottesville VA 22904-4182, cmo3h@virginia.edu. The first draft of this paper, where the application and the methodology were quite different, initially circulated under the title "Time-Varying European Business Cycles". We thank participants at the Euro Area Business Cycle Network (Madrid), 2003 Midwest Macro, Fall 2003 Macro System Committee, 2003 Econometric Society, Cleveland Fed Empirical Methods Conference (2006), 2006 ESEM, St Louis Fed Econometrics Conference (2007), University of Pennsylvania, the European Central Bank, the Bank of England as well as Filippo Altissimo, Marcelle Chauvet, Simon Potter, Morten Ravn, Chris Sims, Dan Waggoner, and Tao Zha for helpful comments. All mistakes are ours. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of New York, or the Federal Reserve System.

1 Introduction

Over the past two decades dynamic factor models have become a standard econometric tool for both measuring comovement in and forecasting macroeconomic time series. The popularity of these models has risen as methods have been developed to perform factor analysis on large datasets, such as the time-domain approach of Stock and Watson (2002a) and the frequency-domain approach of Forni and Reichlin (1998) and Forni, Hallin, Lippi, Reichlin (2001, 2005).[1] The work of Otrok and Whiteman (1998) and Kim and Nelson (1999b) provides a Bayesian alternative to the classical approaches. One goal of this literature has been to extract information from large datasets that is useful in forecasting exercises. A second goal, and the one we focus on this paper, is the use of the factor models to quantify both the extent and nature of comovement in a set of time series data.[2] [3]

An assumption of most dynamic factor models is that both the stochastic process driving volatility and the nature of comovement among variables has not changed over time.[4] But much recent empirical work shows that the assumption of structural stability is invalid for many macroeconomic datasets. For example, McConnell and Perez-Quiros (2000) show that output volatility in the US has moderated since the mid 1980s, a fact known as 'the great moderation'. In a methodological contrast to the break test approach of most of the great moderation literature, Cogley and Sargent (2001) show how the nature of inflation dynamics has evolved in the post war period using a time-varying parameter model to model a more

[1]Bai and Ng (2002) provide the econometric theory for choosing the optimal number of factors in these large scale models.

[2]Forni and Reichlin (1998) study the role of sector specific and aggregate technological shocks on disaggregated industry level output data in the US. Stock and Watson (1999) use their model to study the dynamics of inflation in the US using sectoral inflation data. In the international context Forni and Reichlin (2001) study comovement of regional output in European countries. Kose, Otrok and Whiteman (2003) quantify the importance of world, regional and country specific cycles in international macroeconomic data.

[3]Factor models have a direct mapping in dynamic stochastic general equilibrium models (DSGE) where the observables respond to common unobserved state variables (e.g. Altug 1989, Sargent 1989). The dynamic factor model uses many noisy signals of the observable data to extract information about the underlying structural sources of comovement, and provide empirical evidence on the nature of macroeconomic fluctuations that can be used to inform the building of structural models. The model developed here provides a bridge to the recent literature investigating changes in volatility in a DSGE model (e.g. Justiniano and Primiceri 2007).

[4]Chauvet and Potter (2001) represents an exception, as they estimate a regime-switching factor model on four variables. Mumtaz and Surico (2006) also estimate a factor model with some time-variation in the parameters, following the motivation and to some extent the approach outlined in the first draft of this paper (Del Negro and Otrok 2003).

slowly changing dynamics in the data. The message of all of this work is that models with fixed parameters may not do too well at describing macroeconomic data.

In this paper we develop a dynamic factor model with time-varying coefficients and stochastic volatility to bridge the literature on factor models with the literature on parameter instability. This measurement tool is designed to capture changing comovement among time series by allowing for their dependence on common factors to evolve over time. It also allows for stochastic volatility in the innovations to the processes followed by the factors and the idiosyncratic components. Time variation in the factor loadings and stochastic volatilities are modeled as a random walk, following Cogley and Sargent's (2000) work on vector autoregressions.[5] The estimation procedure is Bayesian and parametric and employs a Gibbs sampler techniques to draw from the exact finite sample joint posterior distribution of the parameters and factors.

We apply our time-varying dynamic factor model to study international business cycle dynamics in the post-Bretton Wood period, using data on the growth rates of real output for 19 developed countries. Recent work documents that international business cycles changed in two important dimensions during this period. First, Blanchard and Simon (2001) show that there has been a global decline in output volatility in G7 countries, with the magnitude and timing of the decline differing across countries (see also Summers, 2005).[6] Second, some papers claim that the nature of comovement across G7 countries has also changed over time (e.g., Heathcote and Perri, 2004). We use our measurement tool to document changes in both volatility and synchronization. The factor structure captures comovement, both at the global level (e.g., due to common shocks to technology as in the international business cycle literature, or to spillovers from one country to the rest of the world) and at the regional one (e.g., European cycles). Changes in the volatility of the factors allow for variation in the importance of global or regional shocks. We also allow the sensitivity (factor loading)

[5] Recent work by Stock and Watson (2007) provides "smoothness" conditions under which the principal component estimates of latent factors are still consistent under parameter instability. However, when time variation is relatively sudden, or is large, these conditions may not be met. Most importantly, their approach does not allow one to identify and measure the timing and extent of the changes, which is the objective of this paper, but rather shows that when these changes are small, and smooth, non-parametric methods may still be robust.

[6] While there is consensus that volatility in the US did decline, there is generally no agreement in the literature on whether this phenomenon is the outcome of structural changes in the economy (e.g. Kahn, McConnell and Gabriel Perez-Quiros 2002, Jaimovich and Shin 2007, Gali and Gambetti 2007), good policy (e.g. Blanchard and Simon 2001), or good luck as manifested by smaller shocks (e.g. Stock and Watson 2002b, Ahmed, Levin and Wilson 2004, Sims and Zha 2006). Stock and Watson (2002) and Giannone, Lenza, and Reichlin (2008) provide a review of the literature.

of each country to the common factors to vary over time, reflecting the possibility that some countries may have become more (or less) sensitive to global shocks due to changes in policy or in the structure of the economy (e.g., financial or trade integration). Finally, the standard deviation of country-specific shocks can also evolve.

In contrast with much of the previous literature, our approach does not rely on rolling window/subsample estimates, or univariate analysis: We use a methodology that explicitly models time-variation in the coefficients and, at the same time, considers all countries jointly. We can therefore present statistical evidence on the timing and the extent of the decline in volatility for different countries taking into account that the sources of this decline can be both international and domestic. Since we do not impose that parameters change for all countries at the same time, as customary in the break test literature, the framework can accommodate the cross-country heterogeneity first shown by Blanchard and Simon (2001).

We find that for most countries the variance of output growth has declined significantly since the 1970s, but that there is much heterogeneity in the magnitude, timing, and source of the decline in volatility. The international business cycle component played a role in the great moderation, especially for G7 countries, but rarely in isolation: For most countries the variance attributed to both domestic and international cycles drops. Countries' sensitivities to international cycles evolve differently over time, however. For the US in particular, the sensitivity to international business cycles and domestic business cycles decline sharply roughly in the same period, the eighties. For Japan, the sensitivity to international business cycle also declines, albeit mostly in the seventies, but the variance of domestic cycles does not. These findings indicates that a decline in the magnitude of international shocks cannot be the only explanation for the decline in volatility.

We find statistical evidence that the average cross-country correlation among G7 countries has declined over time, in line with Heathcote and Perri (2004). The average correlation across all countries, European countries, or Euro area countries has remained the same since 1970, however. Consistent with these findings, there is little statistical evidence that the relative importance of international and domestic business cycles in the variance decomposition has changed in the post-Bretton Wood period, except for some of the G7 countries. In sum, we find no evidence that higher financial and trade integration increases the correlation of business-cycles across countries.

Finally, we document a decline in the cross-sectional dispersion of output volatility across countries, which to our knowledge is a novel finding. In other words, we show that since the Bretton Woods period there has been a convergence across countries in terms of the magnitude of business cycle fluctuations. This is true for G7 countries, as well as for all the countries in the sample.

The rest of the paper is divided into three sections. Section 2 describes the structure of the model. Section 3 briefly discusses the Gibbs sampler we use for model estimation (more details are provided in the technical appendix). Readers not interested in the econometric implementation of the model can skip this section. The results are presented and discussed in section 3.

2 The Model

Dynamic factor models decompose the dynamics of observables $y_{i,t},\ i = 1,\dots,n,\ t = 1,\dots,T$ into the sum of two unobservable components, one that affects all y_is, namely the factors f_t, and one that is idiosyncratic, e.g. specific to each i:

$$y_{i,t} = a_i + b_i f_t + \epsilon_{i,t}, \tag{1}$$

where a_i is the constant and b_i is the exposure, or loading, of series i to the common factors. Although the setup accommodates multiple factors, and indeed we have two factors in the empirical application – for clarity of exposition in this section we write the equations as if we had only one factor. Both the factor and idiosyncratic components follow autoregressive processes of order q and p_i respectively:

$$f_t = \phi_{0,1} f_{t-1} + .. + \phi_{0,q} f_{t-q} + u_{0,t}, \text{ and} \tag{2}$$

$$\epsilon_{i,t} = \phi_{i,1}\epsilon_{i,t-1} + .. + \phi_{i,p_i}\epsilon_{i,t-p_i} + \sigma_i u_{i,t}, \tag{3}$$

where σ_i is the standard deviation of the idiosyncratic component, and $u_{i,t} \sim \mathcal{N}(0,1)$ for $i = 0$ and $i = 1,..,n$ are the innovations to the law of motions 2 and 3, respectively.[7] These innovations are i.i.d. over time and across i. The latter is the key identifying assumption in the model, as it postulates that all comovement in the data arises from the factors.[8] The factors' innovations are also assumed to be uncorrelated with one another. Expressions (1), (2), (3) can be viewed as the measurement and transition equations, respectively,

[7] Note that σ_0 is set to 1, which is a standard normalization assumption given that the scale of the loadings b_is and σ_0 cannot be separately identified.

[8] The literature has also considered "approximate" factor models, that is, models where the idiosyncratic shocks can be cross-sectionally correlated. Doz, Giannone and Reichlin (2006) show that even in this situation maximum likelihood asymptotically delivers consistent estimates of the factors. This result is important for us since we effectively use maximum likelihood techniques.

in a state-space representation. The model just described is the standard dynamic factor model estimated for example in Stock and Watson (1989), and used to study international business cycles in Kose et al. (2003).

We modify the standard factor model in three ways. First, we make the loadings time-varying. This feature allows for changes in the sensitivity of individual series to common factors. Our measurement equation then becomes:

$$y_{i,t} = a_i + b_{i,t} f_t + \epsilon_{i,t} \tag{4}$$

where we control the evolution of the factor loadings by requiring that they follow a random walk without drift:

$$b_{i,t} = b_{i,t-1} + \sigma_{\eta_i} \eta_{i,t}. \tag{5}$$

We assume that $\eta_{i,t} \sim \mathcal{N}(0,1)$ and is independent across i. This is a stark, but important, identifying assumption as we will see later.

The second innovation amounts to introducing stochastic volatility in the law of motion of the factors and the idiosyncratic shocks. The transition equations, (2) and (3) become:

$$f_t = \phi_{0,1} f_{t-1} + .. + \phi_{0,q} f_{t-q} + e^{h_{0,t}} u_{0,t} \quad \text{,and} \tag{6}$$

$$\epsilon_{i,t} = \phi_{i,1} \epsilon_{i,t-1} + .. + \phi_{i,p_i} \epsilon_{i,t-p_i} + \sigma_i e^{h_{i,t}} u_{i,t}, \tag{7}$$

respectively. The terms $e^{h_{i,t}}$ represents the stochastic volatility components, where $h_{i,t}$ follows a random walk process:

$$h_{i,t} = h_{i,t-1} + \sigma_{\zeta_i} \zeta_{i,t}, \;\; i = 0, \;\; 1, .., n \tag{8}$$

with $\zeta_{i,t} \sim \mathcal{N}(0,1)$ and is independent across i (note that $h_{0,t}$ denotes the factor's stochastic volatility term). We assume that $h_{i,t} = 0$ for $t \leq 0, \; i = 0, 1, ., n$, that is, before the beginning of the sample period there is no stochastic volatility: This assumption allows us to derive an ergodic distribution for the initial conditions. In the remainder of the section we will sometimes use the notation: $\sigma_{i,t} = \sigma_i e^{h_{i,t}}$.

2.1 Normalization, identification, and modeling choices

Several issues of identification and normalization arise in this model. First, in (4) both the loadings and the factors vary over time. However, the assumption that the $\eta_{i,t}$s are independent across i implies that only the factors capture comovement among the series.

Second, a normalization issue is that the relative scale of loadings and factors is indeterminate. We can multiply $b_{i,t}$ by κ for all i, obtaining $\tilde{b}_{i,t} = \kappa b_{i,t}$, and divide the factor f_t by κ, obtaining $\tilde{f}_t = f_t/\kappa$. As long as the standard deviation of the innovation in 6 is adjusted accordingly as $e^{\tilde{h}_{0,t}} = e^{h_{0,t} - log(\kappa)}$, the two models are observationally equivalent. This is a standard problem, even when parameters do not vary over time, and we address it by constraining the scale of the factor (see footnote 7). The scale of $h_{0,t}$ depends on the initial condition $h_{0,0}$. We therefore fix $h_{0,0} = 0$. Parameterizing the volatility of the innovations to the idiosyncratic shocks $\sigma_{i,t}$ as the product of σ_i and $e^{h_{i,t}}$ induces a related normalization problem: $\sigma_i e^{h_{i,t}} = \frac{\sigma_i}{\kappa} e^{h_{i,t} + log(\kappa)}$. This problem is similarly addressed by fixing the initial value of the process, $h_{i,0} = 0$ for all i.

Third, given time variation in the loadings and in the factor's volatilities, we need to worry about the possibility that the rescaling is time-varying, that is, that the rescaling term is given by a sequence $\{\kappa_t\}_{t=1}^T$ instead of a scalar κ. Specifically, one could rescale the loadings $b_{i,t}$ for all i by κ_t, $b_{i,t-1}$ by κ_{t-1}, and so on. This normalization problem is avoided because: i) since the factor follow an AR(q) process, the rescaled factors $\tilde{f}_t = f_t/\kappa_t$ would not satisfy (6); ii) the rescaled loadings $\tilde{b}_{i,t} = \kappa_t b_{i,t}$ would not satisfy (5). Even if the strict normalization problem is avoided, one could still be concerned that the identification of "common" shifts in the loadings from changes in the stochastic volatility of the factor is tenuous. The answer is simple: common shifts in the loadings are ruled out from the start by assuming that the innovations in (5) are independent across i. In other words, if the data were to indicate that the exposures have shifted for all i, the model would capture this phenomenon by changes in the stochastic volatility of the factor.

A final normalization issue common to all factor models is that the sign of the factor f_t and the loadings $b_{i,t}$ is indeterminate: the likelihood is the same if we multiply both f_t and the $b_{i,t}$s, for *all* i, by -1. In practice this is a problem to the extent that the MCMC draws switch sign along the Markov chain. We did not find this to be a problem in our application.[9] In applications where this is an issue, Hamilton, Waggoner, and Zha (2007) provide a solution.

We conclude the section with a discussion of modeling choices. We modeled time-variation in both loadings and stochastic volatilities as a drift process as in Cogley and Sargent (2001, 2005). The belief that changes, whether due to policy or structure or luck, are permanent rather than transitory leads us to to prefer this specification over a stationary process. Another alternative consists in modeling time-variation as the result of switching

[9]We use the cross-sectional mean of the data to initialize the factor in the MCMC procedure.

across regimes, as in Sims and Zha (2006), or as structural breaks as in Doyle and Faust (2005). The evidence on cross-country heterogeneity in the timing of the great moderation poses a modeling challenge for this approach: to capture this heterogeneity we would need several regimes. We leave this interesting challenge to future research.

The only non time-varying parameters in our setup are those of the law of motion of the factors and the idiosyncratic shocks (equations 6 and 7). From a computational standpoint making these parameters time-varying would not be a challenge (see Mumtaz and Surico 2006). In this paper, we do not follow this route for two reasons: First, time-variation in these parameters would raise additional identification issues. Second, evidence in Ahmed et al. (2005) indicates that there is little evidence of changes in the shape of the normalized spectrum for US output growth rates; that is, there is no compelling evidence in favor of time-variation for these parameters. Again, we leave to further research to test whether the dynamics of the factors or the idiosyncratic shocks have changed.

2.2 Our Application: World Business Cycles and the Great Moderation

So far we described the time-varying parameters factor model in fairly general terms. In this section we discuss the equations and the assumptions in the context of the specific application pursued in this paper: the study of international business cycles. The measurement equation we use is:

$$y_{i,t} = a_i + b^w_{i,t} f^w_t + b^e_{i,t} f^e_t + \epsilon_{i,t}, \tag{9}$$

where f^w_t and f^e_t represent the world and the European factor, respectively, and where the idiosyncratic terms $\epsilon_{i,t}$ can be interpreted as the combination of two effects: country-specific shocks and, potentially, measurement error. The two factors are separately identified by the assumption that the loadings $b^e_{i,t}$ are set to zero for non-European countries.[10] The evolution of each of the factors and of the idiosyncratic terms are described by equations (6) and (7), where we set $q = 3$ and $p_i = 2$. We should stress that common factors capture not only common shocks, but more in general comovement across countries, which could be due to spillovers from one country (or set of countries) to the rest of the world.

The evolution of the factor loadings and of the stochastic volatilities are described by equations (5) and (8). In these equations we assume that the innovations in the factor load-

[10] This approach to identifying "regional" shocks is standard in this literature: see Kose et al. (2003) and Del Negro and Otrok (2007). Also, we have only one world factor following the results in Kose et al. (2003).

ings for the world and the European factors, called $\eta^w_{i,t}$ and $\eta^e_{i,t}$ respectively, are independent from each other. [11] Likewise we assume that the stochastic volatilities for the world and the European factors, $\zeta^w_{0,t}$ and $\zeta^e_{0,t}$, are also independent from each other.

In our application the drifts in the volatility of the factors, $h^w_{0,t}$ (world) and $h^e_{0,t}$ (European), can capture two distinct phenomena: one is changes in the sensitivity to global conditions that are common across countries, which may occur if all countries undergo the same policy/structural transformations; the other is changes in the volatility of international shocks (say, in commodity price shocks). Changes in the factor loadings $b^w_{i,t}$ (world) and $b^e_{i,t}$ over time may stem from policies adopted by specific countries, or from structural changes such as increased trade or financial integration of a country with the rest of the world (or Europe). As we stressed in the previous section, the assumption that changes in the loadings are independent across i implies that these changes are country-specific in nature. Likewise, drifts in the volatility of the idiosyncratic component can be due to policy/structural transformations that make countries more or less sensitive to domestic shocks, or can be exogenous.

2.3 Priors

Given the paper's question – Is there any time-variation? What are its origins? – the key priors are those for the standard deviations of the innovations to the law of motions of the loadings ($\{\sigma_{\eta_i}\}_{i=1}^n$) and stochastic volatilities ($\{\sigma_{\zeta_i}\}_{i=0}^n$, where $i = 0$ denotes the stochastic volatility for the factors' law of motion). Our prior for the factor loading and stochastic volatility innovations embodies the view that both loadings and volatilities evolve slowly over time and pick up permanent, trend changes, in the economy. Cyclical variation is designed to be captured by the factors and idiosyncratic terms.

The prior distribution for σ_{η_i} is an inverse gamma $IG(\nu_{\eta_i}, s^2_{\eta_i})$, that is:

$$p(\sigma_{\eta_i}|\nu_{\eta_i}, s^2_{\eta_i}) = \frac{2}{\Gamma(\nu_{\eta_i}/2)} \left(\frac{\nu_{\eta_i}}{2} s^2_{\eta_i}\right)^{\frac{\nu_{\eta_i}}{2}} (\sigma^2_{\eta_i})^{-\frac{\nu_{\eta_i}}{2}-\frac{1}{2}} exp\{-\frac{\nu_{\eta_i}}{2}\frac{s^2_{\eta_i}}{\sigma^2_{\eta_i}}\}. \tag{10}$$

Since this prior is conjugate it can be interpreted as having $t^* = 1, .., \nu_{\eta_i}$ fictitious observations of the innovations to equation (5), such that their average squared sum $\frac{1}{\nu_{\eta_i}} \sum_{t^*=1}^{\nu_{\eta_i}} (b_{i,t^*} - b_{i,t^*-1})^2$ equals $s^2_{\eta_i}$. Hence the prior embodies the belief that the variance of the innovations equals $s^2_{\eta_i}$, where the strength of the belief increases proportionally with ν_{η_i}. Similarly, the

[11] Note that this is by no mean a key identifying assumption, as it is the assumptions that the $\eta_{i,t}$s are independent across i, and can in principle be relaxed. We chose to keep the $\eta^w_{0,t}$ and $\eta^e_{0,t}$ uncorrelated to avoid introducing additional free parameters.

prior distribution for σ_{ζ_i} is an inverse gamma $IG(\nu_{\zeta_i}, s^2_{\zeta_i})$. We express ν_{η_i} and ν_{ζ_i} as a fraction of the sample length T to make the weight of the prior relative to the sample explicit. Our prior for σ_{ζ_i} is the same for all idiosyncratic terms ($i = 1, .., n$) and for the factor ($i = 0$). In our baseline prior we set $\nu_{\eta_i} = .1 \times T$, $s^2_{\eta_i} = (.1^2)$, $\nu_{\zeta_i} = T$, and $s^2_{\zeta_i} = (.025^2)$. Hence our baseline prior embodies the belief that the amount of time variation in the stochastic volatilities is smaller relative to that in the loadings. Moreover, our belief is considerably tighter for the volatilities than for the loadings. We use this prior precisely because it stacks the deck against finding strong movements in the stochastic volatilities, and in favor of finding time-variation in the loadings. We checked however for robustness to tighter beliefs on the time variation in the loadings ($\nu_{\eta_i} = .25 \times T, .5 \times T, T$ and $s^2_{\eta_i} = (.05^2)$ and looser ones on the on the time variation in the volatilities ($\nu_{\zeta_i} = .5 \times T$). While not surprisingly under the alternative priors the loadings are even less time-varying than under the baseline specification, qualitatively the results do not change.

The priors for the remaining parameters are as follows. The prior for the non time-varying part of the idiosyncratic volatility σ_i is also given by an inverse gamma distribution: $IG(\nu_i, s_i^2)$. The parameters are chosen so that this prior is quite agnostic: $\nu_i = .05 \times T$ and $s_i = 1$. The priors on the constant terms are normal $a_i : N_k(\bar{a}_i, \bar{A}_i^{-1})$, with mean $\bar{a}_i = 2$ (the growth rates are annualized) and precision $\bar{A}_i = 1$. The prior distribution for the initial conditions for the loadings, $b_{i,0}$, are also normal: $\beta_i : N_k(\bar{\beta}_i, \bar{B}_i^{-1})$, with mean $\bar{\beta}_i = 0$ and precision $\bar{B}_i = 1/10$. The autoregressive coefficients for the factors and the idiosyncratic shocks have a normal prior: $\phi_i : N_k(\bar{\phi}_i, \bar{V}_i^{-1}) I_{S\phi_i}$, where $\phi_i = (\phi_{i,1}, .., \phi_{i,p_i})$, and $I_{S\phi_i}$ is an indicator function that places zero prior mass on the region of the parameter space where things are non stationary. As in Kose, Otrok, and Whiteman (2003) the prior is centered at $\bar{\phi}_i = \{0, 0, .., 0\}'$, thereby favoring parsimonious specifications. The precision matrix for the factor's autoregressive coefficients $\bar{V}_0$ is diagonal with elements proportional to $1/(.75)^l$, where l is the lag length. The precision matrix for the idiosyncratic terms' autoregressive coefficients $\bar{V}_i$ is looser, being equal to $.2 \times \bar{V}_0$. All priors are mutually independent.

3 Estimation

The model is estimated using a Gibbs sampling procedure. In essence, this amounts to reducing a complex problem – sampling from the joint posterior distribution – into a sequence of tractable ones for which the literature already provides a solution – sampling from conditional distributions for a subset of the parameters conditional on all the other parameters.

Our Gibbs sampling procedure reduces to four main blocks. In the first block we condition on the factors, time-varying loadings, stochastic volatilities, and sample from the posterior of the constant term a_i, the autoregressive parameters $\{\phi_{i,1}, \ldots, \phi_{i,p_i}\}$, and the non time-varying component of the variance σ_i^2. Next, we draw the factors f_t conditional on all other parameters using the state space representation of the model, as in Carter and Kohn (1994). The third block draws the time-varying loadings $b_{i,t}$, again using Carter and Kohn's algorithm. In this block the factors are treated as known quantities. In the last block we sample the stochastic volatilities using the procedure of Kim, Sheppard and Chib (1998). In the remainder of the section we intuitively discuss how we apply existing techniques in the literature to derive the conditional distributions for each block of the Gibbs sampler. We will stress that the procedure is computationally efficient and can accommodate datasets where T and n are relatively large: in most cases the computational cost increases only linearly in these dimensions. Again, for the sake of exposition we describe the model as if we had just one common factor. The full derivation of these distributions are provided in the technical appendix. We use 22000 draws, and discard the first 2000, in the actual implementation of the Gibbs sampler. We check for convergence by running several replications and comparing the results.

3.1 Block I: non time-varying parameters

In the first block of the Gibbs Sampler we condition on the factors, the time-varying loadings, and the stochastic volatilities and draw the regression parameters that are not time varying, namely the mean a_i, the autoregressive coefficients $\{\phi_{i,1}, \ldots, \phi_{i,p_i}\}$ of the idiosyncratic component and the non time-varying component of the variance σ_i^2. The key insight from Otrok and Whiteman (1998) is that, conditional on the factors, equation (4) is simply a regression where the errors follow an AR(p) process given by equation (7). Hence drawing from the conditional distribution of a_i, $\{\phi_{i,1}, \ldots, \phi_{i,p_i}\}$, and σ_i^2 amounts to straightforward application in Chib and Greenberg (1994). Moreover, conditional on the factors the errors in equation (4) are independent across i. Therefore the procedure can be implemented one i at the time, which implies that computational cost is linear in the size of the cross-section. The presence of stochastic volatility requires a simple modification of the Chib and Greenberg (1994) procedure: Since the errors in (7) are now heteroskedastic, the posterior distribution of the parameters of interest now involves a GLS correction. For the sake of exposition we leave the details of this modification to the appendix (section A.1).

The autoregressive terms in the law of motion of the factors (7) represent another set of non time-varying parameters. Since we are conditioning on the factors we can again apply the Chib and Greenberg (1994) procedure as described in Otrok and Whiteman (1998). Because of stochastic volatility we again need to correct for heteroskedasticity. Section A.2 in the appendix provides the details.

Finally, we derive the posterior distributions for the standard deviations of the innovations to the law of motions of the loadings σ_{η_i} conditional on the loadings. Given our assumptions that the shocks to equation (5) are normal, and that the prior is conjugate, it follows that this distribution is an inverse gamma $IG(\nu_{\eta_i} + T, \frac{\nu_{\eta_i}\delta^2_{\eta_i} + T d_i^2}{\nu_{\eta_i} + T})$ where $d_i^2 = \frac{1}{T}\sum_{t=1}^{T}(b_{i,t} - b_{i,t-1})^2$. Similarly, conditional on the stochastic volatilities the posterior distributions for σ_{ζ_i} follows an inverse gamma $IG(\nu_{\zeta_i} + T, \frac{\nu_{\zeta_i}\delta^2_{\zeta_i} + T c_i^2}{\nu_{\zeta_i} + T})$ where $c_i^2 = \frac{1}{T}\sum_{t=1}^{T}(h_{i,t} - h_{i,t-1})^2$. This applies to the idiosyncratic terms ($i = 1, .., n$) and to the factors ($i = 0$).

3.2 Block II: factors

In the second block of the Gibbs sampler we draw the factors conditional on all other parameters. Two approaches to drawing the factors exist in the Bayesian factor model literature. One is the Otrok and Whiteman (1998) approach, which builds on Chib and Greenberg (1994) to explicitly write the likelihood of the observations $y_{i,t}$, the prior for the factors (given by equation 6), and then combining the two to derive the posterior distribution of $\{f_t\}_{t=1}^{T}$. The alternative is to use the state-space representation given by equations (4), (6), and (7), and apply the algorithm developed by Carter and Kohn (1994) to draw from the posterior distribution of the factors. The first approach involves inverting size-T matrices, and hence becomes computationally expensive for large T. In the second approach the dimension of the state vector increases with n, since the idiosyncratic terms in (4) are not i.i.d. Hence this approach is computationally unfeasible for large cross-sections. Therefore it becomes essential to eliminate the idiosyncratic terms from the state vector if one wants to estimate data sets where both T and n are potentially large. This is accomplished by quasi-differencing expression (4) as in Kim and Nelson (1999a) and Sargent and Quah (1993).

We follow the second approach. We first write the measurement equation (4) in stacked form:

$$\tilde{y}_t = \tilde{a} + \tilde{b}_t f_t + \tilde{\epsilon}_t \text{ for } t = 1, .., T \tag{11}$$

where $\tilde{y}_t = (y_{1,t}, \ldots, y_{n,t})'$, $\tilde{a} = (a_1, \ldots, a_n)'$, $\tilde{b}_t = (b_{1,t}, \ldots, b_{n,t})'$, $\tilde{\epsilon}_t = (\epsilon_{1,t}, \ldots, \epsilon_{n,t})'$. Next we express the laws of motion of the factor (equation 6) and of the idiosyncratic shocks (equation 7) in companion form:

$$\tilde{f}_t = \Phi_0 \tilde{f}_{t-1} + \tilde{u}_{0,t}, \tag{12}$$

$$\tilde{\epsilon}_t = \Phi_1 \tilde{\epsilon}_{t-1} + .. + \Phi_p \tilde{\epsilon}_{t-p} + \tilde{u}_t, \tag{13}$$

where $\tilde{f}_t = (f_t, .., f_{t-q+1})'$ and $\tilde{u}_{0,t} = (\sigma_{0,t} u_{0,t}, 0, .., 0)'$ are $q \times 1$ vector, $p = max_{i=1,..,n}(p_i)$, $\tilde{u}_t = (\sigma_{1,t} u_{1,t}, \ldots, \sigma_{n,t} u_{n,t})'$, and the Φ_js are companion matrices:

$$\Phi_0 = \begin{bmatrix} \phi_{0,1}.. & \phi_{0,q} \\ I_{q-1} & 0 \end{bmatrix}, \; \Phi_i = \begin{bmatrix} \phi_{i,1}.. & \phi_{i,p} \\ I_{p-1} & 0 \end{bmatrix}. \tag{14}$$

We consider the case $q - 1 = p$ (which for convenience is the case in our application) and define $\Phi(L) = \sum_{j=1}^{p} \Phi_j L^j$. We pre-multiply expression (11) by the matrix $I_n - \Phi(L)$ and obtain the system:

$$\tilde{y}_t^* = \tilde{a}^* + B_t^* \tilde{f}_t + \tilde{u}_t \text{ for } t = p+1, .., T \tag{15}$$

where $\tilde{y}_t^* = (I_n - \Phi(L))\tilde{y}_t$, $\tilde{a}^* = (I_n - \Phi(L))\tilde{a}$, $\tilde{f}_t = (f_t, .., f_{t-q+1})'$, and:

$$B_t^* = \begin{bmatrix} b_{1,t} & -b_{1,t-1}\phi_{1,1} & \ldots & -b_{1,t-p}\phi_{1,p} \\ \vdots & & \ddots & \vdots \\ b_{n,t} & -b_{n,t-1}\phi_{n,1} & \ldots & -b_{n,t-p}\phi_{n,p} \end{bmatrix}.$$

The errors in the measurement equation (15) are now i.i.d.. We are then ready to draw the factors using the Carter and Kohn (1994) procedure, where the transition equation is given by equation (12). Note that in implementing this procedure the curse of dimensionality does not bite: The relevant magnitude for most computations is the size of the state space $q \times$ (*number of factors*). The sample size T increases the computational burden linearly. The cross-sectional size n only affects the computation of the Kalman gain, as one has to invert an $n \times n$ matrix.

Note that equation (15) naturally starts from $t = p + 1$ since we are using the first p observations for pre-whitening. If we were to condition on these first p observations, as much of the literature does, we would use the mean and variance of the unconditional ergodic distribution of the state vector $\tilde{f}_t$ to initialize the Kalman filter. There may be cases where the initial conditions matter, either because T is not too large (relative to p) or because the factor is very persistent. Hence we provide in section A.3 of the appendix formulas for the mean and variance of the state $\tilde{f}_p$ conditional on the first p observations. We use this mean and variance as our initial condition.

3.3 Block III: time-varying factor loadings

In the third block of the Gibbs sampler we draw the time-varying loadings $\{b_{i,t}\}_{i,t=1}^{n,T}$ conditional on all other parameters and the factors. Again, conditional on the factors the errors in the measurement equation (4) are independent across i. Moreover we assumed that the innovations in the transition equation (5) are independent across i. Consequently, we can draw the time-varying loadings one equation at a time, which implies that the computational cost is again linear in n. We deal with the fact that the errors in the measurement equation (4) are not i.i.d. by applying the quasi differencing operator to each equation, obtaining:

$$y_{i,t}^* = a_i^* + \omega_t^* \tilde{b}_{i,t} + \sigma_{i,t} u_{i,t} \text{ for } t = p_i + 1, .., T \tag{16}$$

where $y_{i,t}^* = (1 - \phi_{i,1}L^1 - .. - \phi_{i,p_i}L^{p_i})y_{i,t}$, $a_i^* = (1 - \phi_{i,1} - .. - \phi_{i,p_i})a$, $\omega_t^* = (f_t, \; -\phi_{i,1}f_{t-1}, \; .., \; -\phi_{i,p_i}f_{t-p_i})$, $\tilde{b}_{i,t} = (b_{i,t}, \ldots, b_{i,t-p_i})'$. The transition equation is simply expression (5) written to accommodate $\tilde{b}_{i,t}$:

$$\tilde{b}_{i,t} = \Xi \tilde{b}_{i,t-1} + \tilde{\eta}_{i,t} \tag{17}$$

where $\tilde{\eta}_{i,t} = (\eta_{i,t}, 0, \ldots, 0)$ and

$$\Xi = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ & I_{p_i} & & 0 \end{bmatrix}.$$

Note that the first t considered in equation (16) is $t = p_i + 1$. As with the case of the factors we derive formulas for the mean and variance of the state $\tilde{b}_{i,p_i}$ conditional on the first p observations (see section A.4 in the appendix).

3.4 Block IV: stochastic volatilities

The final block of the Gibbs partition draws the stochastic volatilities $\{\sigma_{i,t}\}_{i=1,t=1}^{n,T}$ conditional on all other parameters. Our procedure follows the algorithm developed by Kim, Sheppard, and Chib (1998). If we define $z_{i,t}$ as the innovation to the law of motion of the idiosyncratic terms (7):

$$z_{i,t} = \epsilon_{i,t} - \phi_{i,1}\epsilon_{i,t-1} \; .. - \phi_{i,p_i}\epsilon_{i,t-p_i}. \tag{18}$$

by construction we have:

$$z_{i,t} \;\; = \;\; \sigma_i e^{h_{i,t}} u_{i,t}. \tag{19}$$

Conditional on data, factors, loadings, and all other parameters, the $z_{i,t}$ are known quantities for $t \geq p_i + 1$. For $t = 1, .., p_i$ expression (18) involves the terms $(\epsilon_{i,0}, \ldots, \epsilon_{i,1-p_i})$, which we do not have. Therefore we draw these terms using the law of motion (7) (see section A.5 in the appendix). Conditional on these draws all the $z_{i,t}$ are known quantities. At this point we can apply the algorithm in Kim et al. (1998) equations (19) and (8). Section A.6 in the appendix briefly reviews their approach. We can do this one equation at the time given that the shocks to the law of motion of the stochastic volatilities are assumed to be independent across i. The stochastic volatility terms for the factors' law of motion (6) are drawn in an analogous manner (details are in section A.7 of the appendix).

4 Empirical Results

We use our econometric model to study the evolution of international business cycles for developed countries. We focus on three phenomena, two of which have been investigated in the literature, and one that to our knowledge has not been discussed before. The first phenomenon is the great moderation (section 4.2). Stock and Watson (2005) use univariate autoregressions with time-varying coefficients and stochastic volatility to present statistical evidence on the great moderation for G7 countries. Separately, they use a factor-augmented vector-autoregression (VAR) estimated over a pre- and post-1983 subsample to investigate whether international or domestic shocks are the source of the decline in volatility. They find that international shocks played an important role.

The second phenomenon is the change in comovement/correlations across countries (section 4.3). Heathcote and Perri (2004) argue that cross-country correlations have declined since the early 1980s (they plot the correlation between the US and an aggregate of Europe and Japan). Doyle and Faust (2005) estimate a VAR on the GDP growth rates for six countries (G7 minus Japan) imposing three break dates, roughly corresponding to the beginning of each decade (1970-Q1, 1981-Q1, 1992-Q2). They question the statistical significance of the decline in bivariate cross-country correlations. Note that the methodology used by Doyle and Faust (2005) has two limitations relative to the one introduced by this paper. First, the cross-sectional dimension their approach allows for is quite limited, since they use a VAR. Second, the break dates are constrained to be the same for all countries in the sample. Kose, Otrok and Whiteman (2007) estimate a fixed parameter dynamic factor model on three subsamples of the data (1960-1972, 1972-1986, 1986-2003). They conclude that comovement has declined from the period 1972-1986 to the post 1986 sample. In both dimensions, our contribution relative to the existing literature is that we use a flexible specification which does not assume that changes in volatility or synchronization occur for all countries at the same time, and yet considers all countries jointly.

The third phenomenon is the decline in the cross-country dispersion of output volatility across country (section 4.4), which is a novel finding. We show that this convergence in volatility is driven by both the international and the country-specific component of business cycles. We conclude the description of the empirical results by contrasting the US and Japanese experience (section 4.5), and by providing some evidence on the factor model's ability of describing comovement (section 4.6).

4.1 The Data

The data are real GDP for 19 countries: U.S., Canada, Japan, Australia, New Zealand, Austria, U.K., Belgium, Denmark, France, Germany, Ireland, Italy, Netherlands, Norway, Sweden, Switzerland, Finland, Spain. The data are quarterly and cover the period 1970:1-2005:4. The source for the data is the IMF International Financial Statistics database. The level series were converted to one quarter growth rates and then annualized. The German reunification presents a significant outlier in the dataset which would be difficult for any econometric model to match and is considered a one time aberration which is outside of the scope of the model or the regular data generating process. We follow Stock and Watson (2005) and replace the 1991:1 data point with the median of GDP growth in German growth time series.

4.2 The Great Moderation Across Countries

Figure 1 and Table 1 provide evidence on the Great Moderation for the G7 and for all countries in our sample, respectively. For each of the G7 countries the shaded areas in Figure 1 show the variance attributable to international business cycles (black), European business cycles (gray), and country-specific fluctuations (white). For each point in time, each country, and each MCMC draw the variance attributable to international (European) business cycles is computed by multiplying ${\beta^w_{i,t}}^2$ (${\beta^e_{i,t}}^2$) times the variance of the factor, computed from equation (6) using the time t estimate of the factor's stochastic volatility ($h_{0,t}$). Likewise, the variance attributable to country-specific cycles is computed from equation (7) using the time t estimate of the idiosyncratic component's stochastic volatility ($h_{i,t}$). The Figure shows the median across all draws. Given the assumption of independence between factors and country-specific components, the sum of the variance attributed to each component equals the overall variance of output growth for country i at each point in time computed according to the factor model.[12]

Figure 1 shows that while the decline in volatility is a global phenomenon, there is much heterogeneity across countries as to the magnitude, timing, and source of the moderation. In terms of timing, for the US the great moderation largely takes place in the eighties. In contrast for other countries, e.g. the UK, Italy, and Canada, it appears to be a gradual phenomenon beginning in the mid-seventies. These latter results seem at odds with the idea that the great moderation is simply the result of good luck, as many countries initiated their decline in volatility in what is commonly referred to as the period of common shocks. In Japan and France the common shock explanation also appears tenuous, as the decline in volatility in these two countries occurs precisely during the seventies. If changes in the variance of oil shocks, or other common shocks, were the only explanation for why the variance attributed to the world factor has dropped across countries, we would not expect such heterogeneity across countries.

In terms of the magnitude of the drop in volatility the US, UK, and Canada stand out relative to the other G7 countries. In terms of the nature of the decline, for most G7 countries both the volatility attributable to international business cycles and country-specific fluctuations drop. In relative terms, the moderation attributed to the international component of business cycles is particularly important for the US, Canada, and Germany, while in the UK and Italy the moderation is mainly attributed to country specific sources. In Japan and France, conversely, there is no decline in the volatility of the country specific component. Taken together it is difficult to provide a story for the great moderation that is common across countries.

The evidence in Figure 1 is confined to the G7 countries, and does not provide information about the estimation uncertainty (both for expositional reasons). Table 1 completes the picture. The table contains six snapshots of the magnitudes shown in Figure 1, e.g. the variance attributable to world, European, and country-specific cycles, for each country in the sample: 1970-Q2 (beginning of the sample), 1980-Q1, 1985-Q1, 1990-Q1, 1995-Q1, and 2005-Q4 (end of the sample). The line *Total* shows the overall variance, computed as the sum of the variances attributed to each component (we take the sum for each MCMC

[12] We have compared the overall volatilities implied by the factor model shown in Figure 1 with the ten-year rolling window volatilities usually shown in the literature, and found that quantitatively the two are very similar.

draw and show the median value). The figures in parenthesis represent the 90% posterior bands. We test whether for any date the variance (either total or that attributed to each component) is lower than at the beginning of the sample: bold-faced numbers indicate that the variance has significantly declined (at the 10% level) relative to the beginning of the sample. Underlined numbers indicate that the variance has significantly declined (at the 10% level) relative to the previous date.

Table 1 shows that for only five countries out of 19 the decline in overall volatility relative to 1970 is not statistically significant, all of them European: Denmark, Finland, Ireland, Switzerland, and Spain (in Denmark and Spain the volatility significantly declines relative from the eighties to the nineties, however). For the G7 countries the results largely confirm that the findings in Figure 1 are statistically significant. For the US the decline in volatility occurs in the eighties, and it applies to both the international and domestic component. For Japan there is a statistically significant decline in volatility relative to 1970, but it is all due to a decline in the international component during the seventies. Canada, Germany, and France are the only other countries where the decline in the international component is statistically significant. For all other countries the moderation arises from a decline in the importance of the country-specific component.[13]

In summary, we provide statistical evidence supporting Blanchard and Simon's findings that i) the great moderation is a worldwide phenomenon, and ii) there is substantial cross-countries heterogeneity in the timing and magnitude of the moderation. In addition, we show there is much heterogeneity in the source of the moderation, whether domestic or international. This heterogeneity is hard to square with one version of the 'good luck' explanation, namely that the drop in volatility across countries is mainly due to a decline in the volatility of international shocks (common productivity shocks, energy shocks).

4.3 Time-Varying Comovement

The traditional approach to measuring comovement is to calculate pairwise correlations of variables directly from the data. Here we calculate a natural counterpart, the implied pairwise correlation from our model. To do so, at each point in time, and for each MCMC draw, we compute all pair-wise correlations implied by the factor model using the time

[13] For a few countries like New Zealand, Norway, and Ireland, the size of the country specific component at the beginning (New Zealand, Norway) or at the end (Ireland) of the sample is so large that we suspect presence of measurement error. While our methodology can fully accommodate time-variation in the importance of measurement error, we cannot tell it apart from country-specific business cycles.

t estimates of the loadings and stochastic volatilities. Figure 2 plots the (unweighted) average cross-country correlation for four different groups of countries: G7, all countries in our sample, European countries (whether or not part of the European Community), and countries that have joined the Euro. The Figure plots the median and the 90% bands of this average.[14]

Figure 2 shows that on average cross-country correlations have either declined or remained the same over time, depending on the group. For G7 countries the average correlation has declined from a median of .25 at the beginning of the sample to below .1 in 2005. Interestingly, by the end of the sample the 90% bands include zero. Much of the decline (about .1) occurs between the seventies and the eighties, while the remainder occurred during the nineties. The decline is statistically significant: since the early nineties more than 90% of posterior draws indicate that the correlation has declined relative to the beginning of the sample. Heathcote and Perri (2004), who first pointed out the drop in cross-country correlations (their dataset included the US and an aggregate of Europe and Japan), claimed that the phenomenon is related to the increased international financial integration occurred since the mid 1980s. We find that there is some evidence the average correlation has declined since – say – 1984-Q1, but it is not overwhelming: the difference is negative for 83% of posterior draws.

The upper-right panel of Figure 2 shows that the average correlation for the entire set of countries has not changed at all in the past 35 years, indicating that changes in international comovements have been mainly confined to large countries. Interestingly, the same applies to European (lower-left panel) and Euro (lower-right panel) countries. The process of economic and (for Euro countries) monetary integration has brought no discernible change in comovements.

Another interesting feature of Figure 2 is that the average cross-country correlation seems to decrease as we consider groups of countries that, a priori, are more integrated. The median average correlation for "All countries" is .15; the corresponding figure for "European Countries" and "Euro Countries" is .1 and .05, respectively. In principle, this finding is not at odds with economic theory: High capital mobility across countries implies that resources

[14]Our calculation may depart from correlations calculated from the data if we have omitted factors that explain the covariance structure in the data. For example, if there is a common factor between the US and Canada, which we have not included in our model, then our correlation calculation will understate the comovement between the US and Canada. However, we are primarily concerned with changes over time, not the absolute level of comovement. Therefore this issue is only problematic if the omitted factors change in importance over time.

flow at any point in time from the least to the most productive countries, generating a negative correlation in output (see Backus, Kehoe, and Kydland 1993). The theory is harder to reconcile with the time series evidence however: Integration has supposedly increased over time, especially in Europe, but the average cross-country correlation has not declined.

A second approach to characterize comovement is to follow the factor model literature (Kose, Otrok, and Whiteman 2003, 2008) and compute the relative importance of common shocks in explaining the variance of macroeconomic aggregates. Since we have a time-varying model, we can show how this variance decomposition, and hence the degree of comovement, evolves over time. The variance decomposition for variable i takes the factor loading at time t, times the model implied variance at time t of the factor, divided by the model implied variance of the variable itself at time t. Table 2 shows the variance attributable to world, European, and country-specific cycles as a fraction of the total variance for each country in the sample for the same dates as in Table 1: 1970-Q2 (beginning of the sample), 1980-Q1, 1985-Q1, 1990-Q1, 1995-Q1, and 2005-Q4 (end of the sample). All figures are in percent, and numbers in parenthesis represent the 90% posterior bands. We are also interested in testing whether any change in the variance decomposition over time is statistically significant. To this effect, bold-faced figures indicate that the change in the fraction of the variance explained by each component relative to the beginning of the sample is significant at the 10% level. Underlined figures indicate that the change relative to the previous date is significant at the 10% level.

Table 2 shows that for the full set of countries the evidence on changes in the importance of global business cycles is mixed. On average the change is approximately zero. However when considering only the G-7 countries we see an average decline of 12 percent for the world factor. This result is consistent with the subsample analysis in Kose, Otrok and Whiteman (2008), although we find that only for Japan the importance of the world component has shrunk significantly relative to that of the country-specific one. For most other countries there have been no statistically significant significant changes in the variance decomposition. In conclusion, there is little statistical evidence that international business cycles are more or less important now than they were thirty-five years ago, except for G7 countries, some of which have become less integrated, and for a few other countries like Sweden, which have become more integrated. The same applies to the European cycle, which has significantly grown in importance only for the Netherlands. It appears that the wide array of structural changes that took place in Europe the past quarter of a century, namely increased trade and financial integration and coordination of monetary policies, has not resulted in a Euro specific cycle.

4.4 Convergence in the the Volatility of Business Cycles Across Countries

This section argues that there has been another important change in business cycles across developed countries, which unlike the decline in volatility has not been previously been documented: Business cycles have become more similar across countries. More specifically, there has been a convergence in the volatility of fluctuations in activity across developed economies.

Figure 3 shows the evolution over time of the cross-sectional standard deviation in output growth volatility for three groups of countries: G7, all the countries in our sample, and all the countries but those for which volatility at any point in time has been particularly high, namely Norway, New Zealand, and Ireland. Our factor model decomposes volatility into two components, one due to international business cycles and the other due to country-specific fluctuations. We want to understand whether the convergence in volatility is due to one or the other, that is, whether the impact of common shocks is more similar across countries, or whether country-specific cycles have become more similar in magnitude. For each group of countries we therefore show the cross-sectional standard deviation of the volatility attributed to each component.

Specifically, for each MCMC draw, period, and country we compute the model-implied volatility, as described in the previous sections. We then take the cross-sectional standard deviation of volatilities within each group. We show the median across MCMC draws and the 90% bands (shaded area). We are also interested in determining whether the decline in the cross-sectional standard deviation relative to the beginning of the sample is statistically significant. The median is shown as a solid line whenever the decline is significant at the 10% level, and as a dashed line otherwise.

Figure 3 shows the cross-sectional dispersion in volatilities has declined significantly since the 1970s for all groups of countries we consider. The source and the timing of the convergence differ among groups however. For the G7 economies, convergence is largely a side-product of the end of the seventies: from 1980 onward the dispersion barely declines. Also, convergence is mostly due to the impact of international business cycles, which has apparently become more similar across countries. The dispersion due to country-specific fluctuations has also declined for G7 countries, but not as much and not as significantly.

For all countries in the sample, there is a significant decline in the dispersion for both the international and country-specific component. Quantitatively, the latter is by far the most important however. Also, a large component of the decline occurs after the seventies.

One concern for the “All countries” results is that they may be in part driven by measurement error: The volatility of real GDP in Norway and New Zealand at the beginning of the sample, and that of Ireland by the end of the sample, is almost one order of magnitude larger than that of the other countries (see Table 1). For this reason the bottom plots show the time-variation in the cross-sectional dispersion in volatilities without these three countries. Quantitatively this clearly makes a substantial difference. Qualitatively however the decline in the dispersion is still there, and is statistically significant. The decline is of about four percentage points, about the same magnitude as for G7 countries. Unlike for G7 countries, the drop in dispersion is a statistically significant feature both the common and the country-specific component. Moreover, there are differences in the timing of the decline, which for all countries persists beyond the seventies.[15]

4.5 The United States and Japan: A Study in Contrasts

The previous sections focused on the posterior distribution of the objects of interest, namely the time-varying volatilities due to international cycles and country-specific fluctuations, and the cross-country correlations. We have so far not shown the evolution of the underlying parameters of the factor model. Since showing the time-varying volatilities and factor loadings for all 19 countries is unfeasible from an expositional standpoint (the results are available upon request), we will show these parameters for two countries that are of particular interest: the US and Japan.

Figure 4 plots the time-varying loadings to the world factor (top) and standard deviations of the country-specific component (bottom) for the two countries. To highlight the percentage reduction (or increase) in the standard deviations, and the extent to which these are statistically significant, we show the standard deviations relative to the first period, e.g. $\sigma_{i,t}/\sigma_{i,1}$. The solid lines show the median and the shaded areas and dotted lines represent the 68 and 90% bands, respectively.

The interesting feature of the US results is that the exposure to world business cycles and the standard deviation of country-specific shocks both begin their decline at roughly the

[15] In principle one could still argue that perhaps changes in the variance of the measurement error drive these results, even if we do not consider Norway, new Zealand and Ireland. As long as measurement error is country specific, we can rule out this explanation for the international business cycles component.

same time, in the early eighties. Both experience a sharp drop for a decade, and then stay constant or decline modestly for the remainder of the sample. Recall that changes in the exposures capture developments that are specific to the US economy, as opposed to changes in the standard deviation of the world factor, which affect all countries. One interpretation of this finding is that the twin decline is a mere coincidence: The US became less exposed to international shocks just as domestic shocks became less severe. Another interpretation is that changes in policy or in the structure of the economy drive both phenomena. We cannot tell one from the other. But our findings are consistent with the policy/structure view of the great moderation.

Japan's experience stands in contrast to the US one. Japan's economy appears to progressively decouple from the rest of the world from the mid-seventies onward. By the end of the sample its exposure to world business cycles is not statistically different from zero. The decline in the sensitivity to world shocks is not accompanied by a corresponding decline in the sensitivity to domestic ones, as in the US case. Consistently with Figure 1, the variance of country-specific fluctuations stays roughly constant all through the period.

4.6 Does the Factor Model Capture Comovement?

So far we have presented no evidence that what we have been referring to as international business cycles actually captures comovement across countries. The top panel of Figure 5 provides a bird's eye view of the data, as it plots the demeaned real GDP growth rates for the 19 countries in the sample. The decline in output volatility between the first to the second half of the sample is apparent from the picture: Not only in the second half of the sample the large swings that characterize the first half are absent (with the exception of Ireland, represented by the dotted line), but also fluctuations appear to be generally smaller in size. It is also apparent that countries' output growth move together, both during recessions (e.g., in the mid-seventies) and booms (e.g., in the nineties), although this covariation is obscured by the high-frequency component of fluctuations.

The next two panels of Figure 5 use the factor model to decompose these fluctuations into two components: international business cycles (middle panel) and country-specific shocks (lower panel). International business cycles and country specific fluctuations are captured by the terms $b^w_{i,t} f^w_t + b^e_{i,t} f^e_t$ and $\epsilon_{i,t}$ in equation (9), respectively (Figure 5 plots the posterior medians of these magnitudes for each country). Three features emerge from

Figure 5: First, international business cycles are more persistent than country-specific fluctuations. Second, there is a decline in the volatility of both components, consistently with the results shown before. Third, there are no discernible comovements in the idiosyncratic components $\epsilon_{i,t}$, suggesting that the World and European factors indeed capture cycles that are common across countries.

5 Conclusions

The paper developed and estimated a dynamic factor model with time-varying factor loadings and stochastic volatility in the innovations to both the common factors and idiosyncratic components. We use the model as a measurement tool to characterize the evolution of international business cycles since 1970.

There are many other potential applications of our methodology. Within the international business cycle literature the model is well suited to address issues linking trade and financial agreements with the impact this has on both comovement of both real and financial variables. Our model, which explicitly allows for changes in factor loadings is a natural framework to analyze recent policy debates on the supposed 'decoupling' of emerging markets economies.

A second area to apply our model is the forecasting literature, which has long used factor models to improve forecast accuracy. Given the observed evolving dynamics in inflation, interest rates and macroeconomic aggregates we suspect that our model will help to further improve out-of-sample forecast accuracy. The ability of our models' parameters to evolve over time to match changing relationships in the data should be the source of this improvement.

Although in this paper we take our model to macro data, the econometric model also has many applications in finance. Factor models have long been used in that literature for both pricing asset and for portfolio allocation. In a paper similar in spirit to our work, Han(2005) develops a latent factor model with stochastic volatility and shows that the dynamic portfolio strategy suggested by the model outperforms (in risk-return space) other allocation strategies. Our model would allow one to develop dynamic allocation strategies where one would also allow risk sensitivities to assets to vary over time. In the term structure, literature Diebold, Li and Yu (2007) study the properties of the global term structure of interest rates using a latent factor model. Interestingly, they estimate their fixed-parameter constant volatility model on two subsamples of the data and document a changing nature of international yield curve dynamics. Their term structure factor model would be a natural application for our methodology, both because their subsample analysis reveals interesting changes in yield curve dynamics, and because it is natural to use models with time-varying volatility to capture time-varying risk premia.

6 References

Ahmed, Shaghil, Andrew Levin, and Beth Anne Wilson, 2004, "Recent U.S. Macroeconomic Stability: Good Policies, Good Practices, or Good Luck?", The Review of Economics and Statistics, 86(3): 824-832.

Altug, Sumru. 1989. "Time-to-Build and Aggregate Fluctuations: Some New Evidence." International Economic Review 30 (4), 889-920.

Bai, Jushan, and Serena Ng, 2002, "'Determining the Number of factors in Approximate Factor Models,"' Econometrica, vol 70, no 1, pp 191-221.

Blanchard, Olivier and John Simon. 2001. "The Long and Large Decline in U.S. Output Volatility." Brookings Papers on Economic Activity 1, 135-64.

Carter, C.K. and R. Kohn. 1994. "On Gibbs sampling for state space models." Biometrika 81(3), 541-553.

Chib, Siddhartha and Edward Greenberg. 1994. "Bayes inferences in regression models with ARMA(p,q) errors." Journal of Econometrics 64,183-206.

Cogley, Timothy and Thomas J. Sargent. 2001. "Evolving Post-World War II U.S. Inflation Dynamics." NBER Macroeconomics Annual, Volume 16. Cambridge and London: MIT Press, 331-73.

Cogley, Timothy and Thomas J. Sargent. 2005, "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US." Review of Economic Dynamics, 8 (2), 262-302

Chauvet, Marcelle and Simon Potter. 2001. "Recent Changes in the U.S. Business Cycle. " The Manchester School, 69 (5), 481-508.

Del Negro, Marco and Christopher Otrok. 2003. "Time-Varying European Business Cycles." Mimeo, University of Virginia.

Del Negro, Marco and Christopher Otrok. 2007. “99 Luftballons: Monetary policy and the house price boom across U.S. states, ” Journal of Monetary Economics 54 (7), 1962-1985.

Diebold, Francis, Canlin Yi and Vivian Yue. 2007. “Global Yield Curve Dynamics and Interactions: A Generalized Nelson-Siegel Approach,” Journal of Econometrics, Forthcoming.

Doyle, Brian M. and Jon Faust. 2005. “Breaks in the Variability and Comovement of G-7 Economic Growth.” Review of Economics and Statistics, 87 (4), 721-40.

Doz, C., Giannone, D., Reichlin, L., 2006. “A Quasi-Maximum Likelihood Approach For Large Approximate Dynamic Factor Models.” CEPR Discussion Paper 5724.

Forni, Mario, and Lucrezia Reichlin. 1998. “Let’s get real: a factor analytic approach to business cycle dynamics.” Review of Economic Studies (65), 453-473.

Forni, Mario, and Lucrezia Reichlin. 2001. “Federal Policies and Local Economies: Europe and the U.S.” European Economic Review 45, 109-134.

Forni, Mario, Marc Hallin, Marco Lippi, and Lucrezia Reichlin. 2001. “The Generalized Dynamic Factor Model: identification and estimation.” Review of Economics and Statistics 82, 540-554.

Forni, Mario, Marc Hallin, Marco Lippi, and Lucrezia Reichlin. 2005. “The Generalized Dynamic Factor Model: One-Sided Estimation and Forecasting.” Journal of the American Statistical Association 100 (471), 830-40.

Gali, Jordi and Luca Gambetti, 2007, ”‘On The Sources of the Great Moderation,”’ Manuscript.‘

Giannone, Domenico, Michele lenza, Lucrezia Reichlin. 2008. “Explaining the Great Moderation: It Is Not the Shocks.” ECB Working paper series 865.

Hamilton, James D., Daniel F. Waggoner, and Tao Zha. 2007. “Normalization in Econometrics.” Econometric Reviews, 26, 221-252.

Han, Yufeng. 2006. “Asset Allocation with a High Dimensional Latent Factor Stochastic Volatility Model,” Review of Financial Studies, 19, 1.

Heathcote, Jonathan and Fabrizio Perri. 2004. “Financial Globalization and Real Regionalization.” Journal of Economic Theory 119 (1), 207-43.

Jaimovich, Nir, and Henry E. Siu, 2007. "The Young, the Old, and the Restless: Demographics and Business Cycle Volatility." Manuscript. Stanford University and University of British Columbia.

Justiniano, Alejandro and Giorgio E. Primiceri. 2006. "The Time Varying Volatility of Macroeconomic Fluctuations." NBER Working Paper 12022.

Kahn, James A., Margaret M. McConnell, and Gabriel Perez-Quiros, 2002, "On the Causes of the Increased Stability of the U.S. Economy,"' FRBNY Economic Policy Review.

Kim, Chang-Jin, and Charles R. Nelson. 1999a. "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle." Review of Economics and Statistics 81 (4), 608-16.

Kim, Chang-Jin, and Charles R. Nelson. 1999b. "State-space models with regime switching." The MIT Press. Cambridge, Massachusetts.

Kim, Sanjoon, Neil Sheppard and Siddhartha Chib, 1998, "Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models," Review of Economic Studies, 65, 361-393.

Kose, M. Ayhan, Christopher Otrok, and Charles H. Whiteman, 2003. "International Business Cycles: World, Region and Country Specific Factors." American Economic Review, September.

Kose, M. Ayhan, Christopher Otrok, and Charles H. Whiteman, 2007. "Understanding the Evolution of World Business Cycles" Journal of International Economics, forthcoming

McConnell, Margaret M.and Gabriel Perez-Quiros. 2000. "Output Fluctuations in the United States: What Has Changed since the Early 1980's?" American Economic Review 90 (5), 1464-76.

Mumtaz, Haroon and Paolo Surico. 2006. "Evolving International Inflation Dynamics: World and Country Specific Factors." Mimeo, Bank of England.

Otrok, Christopher and Charles H. Whiteman, 1998. "Bayesian leading indicators: measuring and predicting economic conditions in Iowa." International Economic Review 39 (4), 997-1014.

Primiceri, Giorgio E.. 2005. "Time Varying Structural Vector Autoregressions and Monetary Policy" Review of Economic Studies, 72 (3), 821-52.

Quah, D. and T.J. Sargent (1993), "A Dynamic Index Model for Large Cross Sections," in J. Stock and M. Watson (eds.), New Research on Business Cycles, Indicators and Forecasting. New York: National Bureau of Economic Research.

Sargent, Thomas J., 1989, 'Two Models of Measurements and the Investment Accelerator', Journal of Political Economy, vol 97 no 2.

Sims, Christopher A. and Tao Zha. 2006. "Were There Regime Switches in U.S. Monetary Policy?" American Economic Review, 96 (1), 54-81.

Stock, James H., and Mark H. Watson. 1989. "New indices of coincident and leading indicators." NBER Macroeconomics Annual, Cambridge and London: MIT Press, 351-393.

Stock, James H., and Mark H. Watson. 1999. "Forecasting Inflation." Journal of Monetary Economics 44 (2), 293-335.

Stock, James H., and Mark H. Watson. 2002a. "Macroeconomic Forecasting Using Diffusion indexes." Journal of Business and Economic Statistics, Vol2 20 No 2, pp 147-162.

Stock, James H., and Mark H. Watson. 2002b. "Has the Business Cycle Changed and Why?" NBER macroeconomics annual Volume 17. Cambridge and London: MIT Press, 159-218.

Stock, James H., and Mark H. Watson. 2005. "Understanding Changes in International Business Cycle Dynamics." Journal of the European Economic Association, 3 (5), 968-1006.

Stock, James H., and Mark H. Watson. 2007. "Forecasting in Dynamic Factor Models Subject to Structural Instability." Mimeo, Harvard and Princeton Universities.

Summers, Peter M., 2005, "'What Caused the Great Moderation? Some Cross Country Evidence,"' Federal Reserve Bank of Kansas City Economic Review, v 90 no 3.

A Details of the Gibbs Sampler

A.1 Conditional distribution of $\{a_i, \phi_{i,1}, \ldots, \phi_{i,p_i}, \sigma_i^2\}_{i=1}^n$

In this step of the Gibbs Sampler we condition on the factors, the parameters $b_{i,t}$, for $t = 1, .., T;\ i = 1, .., n$, and the stochastic volatilities $h_{i,t}$. The presence of stochastic volatilities forces us to slightly modify the Chib and Greenberg (1994) procedure.

The likelihood of the first p_i observations is more convoluted than in Chib and Greenberg (section **??**). We want to write the joint distribution of $\tilde{\epsilon}_{i,p_i} = [\epsilon_{i,p_i} \ldots \epsilon_{i,1}]'$, where from the measurement equation $\epsilon_{i,t} = y_{i,t} - a_i - b_{i,t} f_t$. Define the companion matrix as:

$$\Phi_i = \begin{bmatrix} \phi_{i,1}.. & \phi_{i,p_i} \\ I_{p_i-1} & 0 \end{bmatrix} \ (p_i \times p_i). \tag{20}$$

We can then write the law of motion of the vector $\tilde{\epsilon}_{i,t} = [\epsilon_{i,t} \ \ldots \ \epsilon_{i,t-p_i+1}]'$ as an AR process in companion form:

$$\tilde{\epsilon}_{i,t} = \Phi_i \tilde{\epsilon}_{i,t-1} + \sigma_{i,t} \ \mathsf{e}_1 \ u_{i,t} \tag{21}$$

where e_1 represents the vector $[1 \ 0 \ \ldots \ 0]'$. We assume that for $t \leq 0$ (that is, before the sample) the idiosyncratic shocks are generated from a constant volatility model, i.e. $h_{i,t} = 0, \sigma_{i,t} = \sigma_i,\ t \leq 0$. Under this assumption the vector: $\tilde{\epsilon}_{i,0} = [\epsilon_{i,0}..\epsilon_{i,-p_i+1}]'$ is generated from a stationary distribution. Hence we can express its unconditional distribution as:

$$\tilde{\epsilon}_{i,0} \sim N(0, \sigma_i^2 \Sigma_i), \tag{22}$$

where Σ_i is the solution of the Lyapunov equation implied by 21 for $t \leq 0$. Using 21 we can also derive the distribution of $\tilde{\epsilon}_{i,p_i} = [\epsilon_{i,p_i}..\epsilon_{i,1}]'$ as:

$$\tilde{\epsilon}_{i,p_i} = \Phi_i^{p_i} \tilde{\epsilon}_{i,0} + \sigma_{i,p_i} \mathsf{e}_1 u_{i,p_i} + \sigma_{i,p_i-1} \Phi_i \mathsf{e}_1 u_{i,p_i-1} + \cdots + \sigma_{i,1} \Phi_i^{p_i-1} \mathsf{e}_1 u_{i,1}. \tag{23}$$

The above equations can be rewritten as:

$$\tilde{\epsilon}_{i,p_i} = \Phi_i^{p_i} \tilde{\epsilon}_{i,0} + \sigma_i \underbrace{\left[e^{h_{i,p_i}} \mathsf{e}_1 \mid e^{h_{i,p_i-1}} \Phi_i \mathsf{e}_1 \mid \ldots \mid e^{h_{i,1}} \Phi_i^{p_i-1} \mathsf{e}_1 \right]}_{\mathcal{Z}_i} \begin{bmatrix} u_{i,p} \\ .. \\ u_{i,2} \\ u_{i,1} \end{bmatrix} \tag{24}$$

Define $\tilde{y}_{i,p_i} = [(y_{i,p_i} - b_{i,p_i} f_{p_i}) \ \ldots \ (y_{i,1} - b_{i,1} f_1)]'$ and $\tilde{x}_{p_i}$ a p_i unit vector. Now define

$$\mathcal{S}_i = \Phi_i^{p_i} \Sigma_i \Phi_i^{p_i \ \prime} + \mathcal{Z}_i \mathcal{Z}_i' \tag{25}$$

and $\mathcal{Q}_i$ the Choleski decomposition of $\mathcal{S}_i$ ($\mathcal{S}_i = \mathcal{Q}_i \mathcal{Q}_i'$). Note that if you use the transformed variables $\tilde{y}_{i,1}^* = \mathcal{Q}_i^{-1} \tilde{y}_{i,1}$ and $\tilde{x}_{i,1}^* = \mathcal{Q}_i^{-1} \tilde{x}_{p_i}$ you can write the likelihood of the first p_i observations, conditional on the factors and the time-varying loadings and variances, as:

$$\begin{aligned} L(\tilde{y}_{i,p_i}|...) &= (2\pi\sigma_i^2)^{-p_i/2}|\mathcal{S}_i|^{-1/2}\exp\{-\tfrac{1}{2\sigma_i^2}(\tilde{y}^*_{i,p_i}-\tilde{x}_{p_i}a_i)'\mathcal{S}_i^{-1}(\tilde{y}^*_{i,p_i}-\tilde{x}_{p_i}a_i)\} \\ &= (2\pi\sigma_i^2)^{-p_i/2}|\mathcal{Q}_i|^{-1}\exp\{-\tfrac{1}{2\sigma_i^2}(\tilde{y}^*_{i,p_i}-\tilde{x}^*_{p_i}a_i)'(\tilde{y}^*_{i,p_i}-\tilde{x}^*_{p_i}a_i)\}. \end{aligned} \tag{26}$$

The likelihood for the last $T-p_i$ observations conditional on the first p_i does not pose any problem. For each i, the likelihood of $y_{i,t}$ conditional on the previous p_i observations is given by:

$$\begin{aligned} L(y_{i,t}|f_t,b_t,..,y_{0,t-p_i},y_{i,t-1},..,y_{i,t-p_i}) \ &= \\ = \ & (2\pi\sigma_{i,t}^2)^{-1/2}\exp\{-\tfrac{1}{2\sigma_{i,t}^2}(y_{i,t}-\hat{y}_{i,t|t-1})^2\} \\ = \ & (2\pi\sigma_i^2 e^{2h_{i,t}})^{-1/2}\exp\{-\tfrac{1}{2\sigma_i^2}\left(\frac{y_{i,t}-\hat{y}_{i,t|t-1}}{e^{h_{i,t}}}\right)^2\} \end{aligned} \tag{27}$$

where $\hat{y}_{i,t|t-1} = a_i + b_{i,t}f_t + \phi_1(y_{i,t-1}-a_i-b_{i,t-1}f_{t-1}) + .. + \phi_{p_i}(y_{i,t-p_i}-a_i-b_{i,t-p_i}f_{t-p_i})$. One can write this expression as a function of either a_i or $\phi_i, i=1,..,p_i$ and derive the posterior of these parameters.

Rearranging 27 we can write the likelihood for the last $T-p_i$ observations can be written (see section **??** for more details) as a function of a_i:

$$L(y_{i,p_i+1},..,y_{i,T}|\varphi,y_{i,1},..,y_{i,p_i},\dots) \propto exp\{-\frac{1}{2\sigma_i^2}(\tilde{y}^*_{i,2}-\tilde{x}^*_{i,2}a_i)'(\tilde{y}^*_{i,2}-\tilde{x}^*_{i,2}a_i)\} \tag{28}$$

where $\tilde{y}^*_{i,2}$ is a $T-p_i\times 1$ vector with elements $\phi_i(L)(y_{i,t}-b_{i,t}f_t)/e^{h_{i,t}}$, and $\tilde{x}^*_{i,2}$ is a $T-p_i\times 1$ matrix whose rows are given by the vector $[\phi_i(1)/e^{h_{i,t}}]$ (essentially we estimate the a_i coefficients by GLS rather than OLS). Define

$$\tilde{y}_i^* = \begin{bmatrix} \tilde{y}^*_{i,1} \\ \tilde{y}^*_{i,2} \end{bmatrix} (T\times 1), \tilde{x}_i^* = \begin{bmatrix} \tilde{x}^*_{i,1} \\ \tilde{x}^*_{i,2} \end{bmatrix} (T\times 2).$$

Combining 29 with 26 and the prior for a_i it is now easy to see that the posterior for a_i, conditional on the factor and all other parameters, is given by:

$$a_i|\dots \sim N_k(A_i^{-1}(\bar{A}_i\bar{a}_i+\sigma_i^{-2}\tilde{x}_i^{*'}\tilde{y}_i^*), A_i^{-1})$$

with $A_i = \bar{A}_i + \sigma_i^{-2}\tilde{x}_i^{*'}\tilde{x}_i^*$. Combining 29 with 26 and the prior for σ_i^2 one obtains the posterior for σ_i^2, conditional on the factor and all other parameters:

$$\sigma_i^2|\dots \sim IG(\frac{\bar{\nu}_i+T}{2}, \bar{\delta}_i^2 + d_i^2)$$

where $d_i^2 = (\tilde{y}_i^* - \tilde{x}_i^* a_i)'(\tilde{y}_i^* - \tilde{x}_i^* a_i)$.

Now we focus on the posterior for $\phi_i, i=1,..,p_i$. Define

$$e_{i,t} = y_{i,t} - a_i - b_{i,t}f_t$$

and note that:

$$y_{i,t} - \hat{y}_{i,t|t-1} = e_{i,t} - \phi_1 e_{i,t-1} ... - \phi_{p_i} e_{i,t-p_i}.$$

This implies that 27 can be expressed as a function of the es and the ϕs:

$$L(y_{i,p_i+1},..,y_{i,T}|\varphi,\tilde{y}_{i,1},\tilde{f}_1) \propto \exp\{-\frac{1}{2\sigma_i^2}(e_i - E_i\phi_i)'(e_i - E_i\phi_i)\} \tag{29}$$

where $e_i = (e_{i,p_i+1}/\sigma_{i,p_i+1},,..,e_{i,T}/\sigma_{i,T})'$ is a $T - p_i \times 1$ vector and E_i is a $T - p_i \times p_i$ matrix whose t^{th} row is given by the $1 \times p_i$ vector $(e_{i,t-1}/\sigma_{i,t},..,e_{i,t-p_i}/\sigma_{i,t})$. Combining 29 with 26 and the prior for ϕ_i one finds the posterior for ϕ_i, conditional on the factor and all other parameters:

$$\phi_i|\cdots \propto \Psi_i(\phi_i) \times N_{p_i}(\Phi_i^{-1}(\bar{\Phi}_i\bar{\phi}_i + \sigma_i^{-2}E_i'e_i),\Phi_i^{-1})I_{S\phi_i}$$

where $\Phi_i = \bar{\Phi}_i + \sigma_i^{-2}E_i'E_i$ and $\Psi_i(\phi_i)$ comes from 26:

$$\Psi_i(\phi_i) = |\mathcal{S}_i|^{-1/2}\exp\{-\frac{1}{2\sigma_i^2}(\tilde{y}^*_{i,p_i} - \tilde{x}_{p_i}a_i)'\mathcal{S}_i^{-1}(\tilde{y}^*_{i,p_i} - \tilde{x}_{p_i}a_i)\}.$$

In the MCMC iteration q we can generate the draw ϕ_i^q from $N_{p_i}(..)I_{S\phi_i}$ and then accept them with probability $\min\left(\frac{\Psi_i(\phi_i^q)}{\Psi_i(\phi_i^{q-1})},1\right)$.

A.2 Conditional distribution of $\{\phi_{0,1},\ldots,\phi_{0,q}\}$

The conditional distribution of the $\phi_{0,q} = \{\phi_{0,1},\ldots,\phi_{0,q}\}'$ can be derived by a straightforward modification of the procedure in Otrok and Whiteman (1998) that takes the stochastic volatilities into account. Note that ϕ_0 does not enter the likelihood - ϕ_0 enters only the prior distribution of the factors. The model 6 implies that such prior is proportional to:

$$\begin{aligned} L(\tilde{f}|\phi_0) &= L(\tilde{f}_1|\phi_0) \times L(\tilde{f}_2|\phi_0) \\ &= (2\pi)^{-p_0/2}|\Sigma_0|^{-1/2}\exp\{-\tfrac{1}{2}\tilde{f}_1'\Sigma_0^{-1}\tilde{f}_1\} \times (2\pi)^{-(T-p_0)/2}\exp\{-\tfrac{1}{2}(e_0 - E_0\phi_0)'(e_0 - E_0\phi_0)\} \end{aligned} \tag{30}$$

where $\tilde{f}_1$ and $\tilde{f}_2$ represent the first p_0 and the last $T - p_0$ elements of $f_t = \{f_1,\ldots,f_T\}$. are defined The posterior for ϕ_0 can therefore be derived as in the previous section:

$$\phi_0|\cdots \propto \Psi_0(\phi_0) \times N_{p_0}(\Phi_0^{-1}(\bar{\Phi}_0\bar{\phi}_0 + E_0'e_0),\Phi_0^{-1})I_{S\phi_0}$$

where $\Phi_0 = \bar{\Phi}_0 + E_0'E_0$, $e_0 = (f_{p_0+1}/\sigma_{0,p_0+1},,..,f_T/\sigma_{0,T})'$ is a $T - p_0 \times 1$ vector and E_0 is a $T - p_0 \times p_0$ matrix whose t^{th} row is given by the $1 \times p_0$ vector $(f_{0,t-1}/\sigma_{0,t},..,f_{0,t-p_0}/\sigma_{0,t})$, and $\Psi_0(\phi_0)$ equals the first part of expression 30.

A.3 Mean and variance of $\tilde{f}_p$ conditional on the first p observations

Define $\tilde{y}^{p..1} = (\tilde{y}_p',\ldots,\tilde{y}_1')'$, and $\tilde{\epsilon}^{p..1} = (\tilde{\epsilon}_p',\ldots,\tilde{\epsilon}_1')'$, and:

$$\bar{B}_t = \begin{bmatrix} b_{1,t} & \\ \vdots & 0_{n,q-1} \\ b_{n,t} & \end{bmatrix}.$$

Note that

$$\tilde{y}_t = \tilde{a} + \bar{B}_t \Phi_0^t \tilde{f}_0 + \bar{B}_t \sum_{j=0}^{t-1} \Phi_0^j \tilde{u}_{0,t-j} + \tilde{\epsilon}_t.$$

Hence we can write the first p observations as:

$$\tilde{y}^{p..1} = \underbrace{\begin{bmatrix} I_n \\ .. \\ I_n \\ I_n \end{bmatrix}}_{\mathcal{I}_y} \tilde{a} + \underbrace{\begin{bmatrix} \bar{B}_p \Phi_0^p \\ .. \\ \bar{B}_2 \Phi_0^2 \\ \bar{B}_1 \Phi_0 \end{bmatrix}}_{\mathcal{B}_y} \tilde{f}_0 + \underbrace{\begin{bmatrix} \bar{B}_p & .. & \bar{B}_p \Phi_0^{p-2} & \bar{B}_p \Phi_0^{p-1} \\ .. & & & \\ 0 & .. & \bar{B}_2 & \bar{B}_2 \Phi_0 \\ 0 & .. & 0 & \bar{B}_1 \end{bmatrix}}_{\mathcal{U}_y} \begin{bmatrix} \tilde{u}_{0,p} \\ .. \\ \tilde{u}_{0,2} \\ \tilde{u}_{0,1} \end{bmatrix} + \tilde{\epsilon}^{p..1}. \tag{31}$$

Moreover

$$\tilde{f}_p = \Phi_0^p \tilde{f}_0 + \underbrace{\begin{bmatrix} I & .. & \Phi_0^{p-2} & \Phi_0^{p-1} \end{bmatrix}}_{\mathcal{U}_f} \begin{bmatrix} \tilde{u}_{0,p} \\ .. \\ \tilde{u}_{0,2} \\ \tilde{u}_{0,1} \end{bmatrix}. \tag{32}$$

Call Σ_0 and $\Sigma_{\epsilon^{p..1}}$ the variance covariance matrix of $\tilde{u}^{p..1} = (\tilde{u}'_{0,p}, \ldots, \tilde{u}'_{0,1})'$ and $\tilde{\epsilon}^{p..1} = (\tilde{\epsilon}'_p, \ldots, \tilde{\epsilon}'_1)'$, respectively. From our distributional assumptions we have that

$$\begin{bmatrix} \tilde{y}^{p..1} \\ \tilde{f}_p \end{bmatrix} = \mathcal{N} \begin{pmatrix} \mathcal{I}_y \tilde{a} + \mathcal{B}_y \tilde{f}_{0,0} & & \mathcal{B}_y \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_y \ \Sigma_0 \ \mathcal{U}'_y + \Sigma_{\epsilon^{p..1}} & ... \\ \Phi_0^p \tilde{f}_{0,0} & , & \Phi_0^p \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_f \ \Sigma_0 \ \mathcal{U}'_y & \Phi_0^p \tilde{s}_{0,0} \Phi_0^{p\,\prime} + \mathcal{U}_f \ \Sigma_0 \ \mathcal{U}'_f \end{pmatrix} \tag{33}$$

where $\tilde{f}_{0,0}$ and $\tilde{s}_{0,0}$ are the unconditional mean and variance of $\tilde{f}_t$. Therefore the conditional mean and the variance of $\tilde{f}_p$ are given by:

$$\begin{aligned} E_{p_i}[\tilde{f}_p] = & \quad \Phi_0^p \tilde{f}_{0,0} + \left(\Phi_0^p \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_f \ \Sigma_0 \ \mathcal{U}'_y\right) \\ & \quad \left(\mathcal{B}_y \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_y \ \Sigma_0 \ \mathcal{U}'_y + \Sigma_{\epsilon^{p..1}}\right)^{-1} \left(\tilde{y}^{p..1} - \mathcal{I}_p \tilde{a} - \mathcal{B}_y \tilde{f}_{0,0}\right) \\ V_{p_i}[\tilde{f}_p] = & \quad \Phi_0^p \tilde{s}_{0,0} \Phi_0^{p\,\prime} + \mathcal{U}_f \ \Sigma_0 \ \mathcal{U}'_f - \left(\Phi_0^p \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_f \ \Sigma_0 \ \mathcal{U}'_y\right) \\ & \quad \left(\mathcal{B}_y \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_y \ \Sigma_0 \ \mathcal{U}'_y + \Sigma_{\epsilon^{p..1}}\right)^{-1} \left(\Phi_0^p \tilde{s}_{0,0} \mathcal{B}'_y + \mathcal{U}_f \ \Sigma_0 \ \mathcal{U}'_y\right)'. \end{aligned} \tag{34}$$

To complete the above formula we need an expression for the matrix $\Sigma_{\epsilon^{p..1}}$, the unconditional variance of $\tilde{\epsilon}^{p..1} = (\tilde{\epsilon}'_p, \ldots, \tilde{\epsilon}'_1)'$. Recall from section A.1 that the unconditional variance of $\tilde{\epsilon}_{i,p} = (\epsilon_{i,p}, .., \epsilon_{i,1})'$ is given by $\sigma_i^2 \mathcal{S}_i$. The vector $\tilde{\epsilon}^{p..1}$ contains $\epsilon_{i,p}$ in row i, $\epsilon_{i,p-1}$ in row $n+i$, and so on. Since the idiosyncratic shocks are uncorrelated across equations the matrix $\Sigma_{\epsilon^{p..1}}$ will then have the following structure:

$$\Sigma_{\epsilon^{p..1}} = \begin{bmatrix} \sigma_{11}^1 & 0 & \ldots & \sigma_{12}^1 & 0 & \ldots & \\ 0 & \sigma_{11}^2 & & 0 & \sigma_{12}^2 & & \\ \vdots & & \ddots & \vdots & & \ddots & \\ \sigma_{21}^1 & 0 & \ldots & & & & \\ 0 & \sigma_{21}^2 & & & \ddots & & \\ \vdots & & \ddots & & & & \end{bmatrix},$$

where σ^i_{kl} the $(k,l)^{\text{th}}$ element of $\mathcal{S}_i$. Finally, Σ_0 is as follows:

$$\Sigma_0 = \begin{bmatrix} \sigma_{0,p} & 0 & & 0 & \dots & & \dots \\ 0 & 0 & & & & & \\ 0 & & & \sigma_{0,p-1} & 0 & & \\ \vdots & & & 0 & 0 & & \\ & & & & & \ddots & \\ \vdots & & & & & & \end{bmatrix},$$

A.4 Mean and variance of $\tilde{b}_{i,p_i}$ conditional on the first p_i observations

Key ingredients in the procedure are the mean and variance for the initial state $\tilde{b}_{i,p_i} = (b_{i,p_i}, \dots, b_{i,1}, b_{i,0})'$ given the first p_i observations $y_i^{p_i..1} = (y_{i,p_i}, \dots, y_{i,1})'$, and the factors. Note that

$$y_{i,t} = a_i + b_{i,0} f_t + (\sum_{j=1}^{t} \eta_{i,j})\ f_t + \epsilon_{i,t}.$$

We can write the first p_i observations as:

$$y_i^{p_i..1} = \underbrace{\begin{bmatrix} 1 \\ .. \\ 1 \\ 1 \end{bmatrix}}_{\mathcal{I}_y} a_i + \underbrace{\begin{bmatrix} f_{p_i} \\ .. \\ f_2 \\ f_1 \end{bmatrix}}_{\mathcal{B}_y} b_{i,0} + \underbrace{\begin{bmatrix} f_{p_i} & .. & f_{p_i} & f_{p_i} \\ .. & & & \\ 0 & .. & f_2 & f_2 \\ 0 & .. & 0 & f_1 \end{bmatrix}}_{\mathcal{U}_y} \begin{bmatrix} \eta_{i,p_i} \\ .. \\ \eta_{i,2} \\ \eta_{i,1} \end{bmatrix} + \begin{bmatrix} \epsilon_{i,p_i} \\ .. \\ \epsilon_{i,2} \\ \epsilon_{i,1} \end{bmatrix} \tag{35}$$

and $\tilde{b}_{i,p_i}$:

$$\tilde{b}_{i,p_i} = \underbrace{\begin{bmatrix} 1 \\ .. \\ .. \\ .. \\ 1 \end{bmatrix}}_{\mathcal{B}_b} b_{i,0} + \underbrace{\begin{bmatrix} 1 & 1 & .. & 1 \\ .. & & & \\ 0 & .. & 1 & 1 \\ 0 & .. & 0 & 1 \\ 0 & .. & 0 & 0 \end{bmatrix}}_{\mathcal{U}_b} \begin{bmatrix} \eta_{i,p_i} \\ .. \\ \eta_{i,2} \\ \eta_{i,1} \end{bmatrix} \tag{36}$$

Call U_y and U_b the two upper triangular matrices in equations 35 and 36, respectively. From our distributional assumptions we have that

$$\begin{bmatrix} y_i^{p_i..1} \\ \tilde{b}_{i,p_i} \end{bmatrix} = \mathcal{N} \begin{pmatrix} \mathcal{I}_y a_i + \mathcal{B}_y \bar{b}_{i,0} & , & \mathcal{B}_y \bar{s}_{i,0} \mathcal{B}_y' + \sigma^2_{\eta_i} U_y\ U_y' + \sigma^2_i \mathcal{S}_i & \dots \\ \mathcal{B}_b \bar{b}_{i,0} & & \mathcal{B}_b \bar{s}_{i,0} \mathcal{B}_y' + \sigma^2_{\eta_i} U_b\ U_y' & \mathcal{B}_b \bar{s}_{i,0} \mathcal{B}_b' + \sigma^2_{\eta_i} U_b\ U_b' \end{pmatrix} \tag{37}$$

where $\bar{b}_{i,0}$ and $\bar{s}_{i,0}$ are the unconditional mean and variance of $b_{i,0}$. Therefore the conditional mean and the variance of $\tilde{b}_{i,p_i}$ are given by:

$$\begin{aligned} E_{p_i}[\tilde{b}_{i,p_i}] &= \mathcal{B}_b \bar{b}_{i,0} \\ &+ (\mathcal{B}_b \bar{s}_{i,0} \mathcal{B}_y' + \sigma_{\eta_i}^2 U_b\ U_y')\ (\mathcal{B}_y \bar{s}_{i,0} \mathcal{B}_y' + \sigma_{\eta_i}^2 U_y\ U_y' + \sigma_i^2 \mathcal{S}_i)^{-1}\ (y_i^{p_i..1} - \mathcal{I}_y a_i - \mathcal{B}_y \bar{b}_{i,0}) \\ V_{p_i}[\tilde{b}_{i,p_i}] &= \mathcal{B}_b \bar{s}_{i,0} \mathcal{B}_b' + \sigma_{\eta_i}^2 U_b\ U_b' \\ &- (\mathcal{B}_b \bar{s}_{i,0} \mathcal{B}_y' + \sigma_{\eta_i}^2 U_b\ U_y')\ (\mathcal{B}_y \bar{s}_{i,0} \mathcal{B}_y' + \sigma_{\eta_i}^2 U_y\ U_y' + \sigma_i^2 \mathcal{S}_i)^{-1}\ (\mathcal{B}_b \bar{s}_{i,0} \mathcal{B}_y' + \sigma_{\eta_i}^2 U_b\ U_y')'. \end{aligned} \tag{38}$$

A.5 Drawing $\{\epsilon_{i,0}, \ldots, \epsilon_{i,1-p_i}\}$

In order to draw $\{\epsilon_{i,0}, \ldots, \epsilon_{i,1-p_i}\}$ we use the law of motion

$$\tilde{\epsilon}_{i,t} = \Phi_i \tilde{\epsilon}_{i,t-1} + \sigma_{i,t}\ [1\ 0\ ...\ 0]'\ u_{i,t}, \tag{39}$$

of the vector $\tilde{\epsilon}_{i,t} = [\epsilon_{i,t}\ \ldots\ \epsilon_{i,t-p_i+1}]'$ (in this step we are using the previous iteration's draw of $\sigma_{i,t}$). Again, $\tilde{\epsilon}_{i,j}$ is known for $j = p_i$. Conditional on $\tilde{\epsilon}_{i,j}$ we can draw $\tilde{\epsilon}_{i,j-1}$ using 39 for $j = p_i, .., 1$. In order to do this we need the unconditional variance of $\tilde{\epsilon}_{i,j}$ and the covariance between $\tilde{\epsilon}_{i,j}$ and $\tilde{\epsilon}_{i,j-1}$ (the unconditional means are all zero). These can be recovered using the recursion:

$$\begin{aligned} Var(\tilde{\epsilon}_{i,j}) &= \Phi_i Var(\tilde{\epsilon}_{i,j-1}) \Phi_i' + \sigma_{i,j}^2\ [1\ 0\ ...\ 0]'\,[1\ 0\ ...\ 0] \\ Cov(\tilde{\epsilon}_{i,j-1}, \tilde{\epsilon}_{i,j}) &= Var(\tilde{\epsilon}_{i,j-1}) \Phi_i' \end{aligned} \tag{40}$$

starting from $Var(\tilde{\epsilon}_{i,0}) = \sigma_{i,0}^2 \Sigma_i$ (see expression 22).

A.6 Drawing the stochastic volatilities

Once we have the $z_{i,t}$s, we can draw the stochastic volatilities using the procedure in Kim, Shephard, and Chib (1998), which we briefly describe. Taking squares and then logs of 19 one obtains:

$$z_{i,t}^* = \log(\sigma_i^2) + 2h_{i,t} + u_{i,t}^* \tag{41}$$

where $z_{i,t}^* = \log(z_{i,t}^2 + c)$, $c = .001$ being the offset constant, and $u_{i,t}^* = \log(u_{i,t}^2)$. If $u_{i,t}^*$ were normally distributed, the $h_{i,t}$ could be drawn as in Carter and Kohn, using 41 as the measurement equation and 8 as the transition equation. In fact, $u_{i,t}^*$ is distributed as a $\log(\chi_1^2)$. Kim, Shephard, and Chib (1998) address this problem by approximating the $\log(\chi_1^2)$ with a mixture of normals, that is, expressing the distribution of $u_{i,t}^*$ as:

$$pdf(u_{i,t}^*) = \sum_{k=1}^{K} q_k \mathcal{N}(m_k^* - 1.2704, \nu_k^{*\ 2}) \tag{42}$$

The parameters that optimize this approximation, namely $\{q_k, m_k^*, \nu_k^*\}_{k=1}^K$ and K, are given in Kim, Shephard, and Chib (1998). Note that these parameters are independent of the specific application. The mixture of normals can be equivalently expressed as:

$$u_{i,t}^* | s_{i,t} = k \sim \mathcal{N}(m_k^* - 1.2704, \nu_k^{*\ 2}),\ Pr(s_{i,t} = k) = q_k. \tag{43}$$

Conditional on $s_{i,t} = k$, the $h_{i,t}$ can be now drawn as in Carter and Kohn (1994), using 41 as the measurement equation and 8 as the transition equation. Therefore, once we have a set of draws for $\{s_{i,t}\}_{t=1}^{T}$, drawing the stochastic volatilities is straightforward.

The next task is to draw the set $\{s_{i,t}\}_{t=1}^{T}$, conditional on the draws for the stochastic volatilities and the other parameters. Conditional on the $h_{i,t}$ and all other parameters, we can use 42 to draw the $s_{i,t}$, i.e.:

$$Pr\{s_{i,t} = k| \dots\} \propto q_k \nu_k^{-1} \exp\{-\frac{1}{2\nu_k^{*\,2}}(u_{i,t}^* - m_k^* + 1.2704)^2\} \tag{44}$$

where from 41 $u_{i,t}^* = z_{i,t}^* - 2h_{i,t}$.

A.7 Drawing the stochastic volatilities for the factor: $\{\sigma_{0,t}\}_{t=1}^{T}$.

Define

$$z_{0,t} = f_t - \phi_{0,1} f_{t-1} \;..- \phi_{0,p_i} f_{t-q}. \tag{45}$$

Using this definition, and the fact that decomposition $\sigma_{0,t} = e^{h_{i,t}}$, expression 6 can be rewritten as:

$$z_{0,t} \;=\; e^{h_{0,t}} u_{0,t}. \tag{46}$$

Since f_t is given at this stage the $z_{0,t}$ are known quantities for $t \geq q$. For $t = 1, .., q$ we need to obtain draws of the idiosyncratic shocks and condition on them. This is done by using further iterations (backward) in the Carter and Kohn procedure. To do this we need the mean and the variance of f_t given the first t observations. For $t = q - 1 = p$ these quantities are obtained in section A.3. A similar approach delivers the same objects for $t = p - 1, .., 1$. Given the $z_{0,t}$s, the remainder of the procedure is as in Kim, Shephard, and Chib (1998).

Table 1: Variance Attributable to World, European, and Idiosyncratic Shocks

Dates		70-Q2		80-Q1		85-Q1		90-Q1		95-Q1		05-Q4	
U.S.	World	12.0	(0.0,26.1)	12.1	(2.4,22.9)	5.8	(1.6,10.9)	**2.6**	(0.5,5.2)	**1.8**	(0.1,3.9)	**1.9**	(0.0,4.9)
	Country	8.7	(3.6,14.2)	7.7	(3.8,12.2)	**5.1**	(2.9,7.5)	**3.7**	(2.3,5.3)	**3.2**	(2.1,4.5)	**2.8**	(1.5,4.3)
	Total	21.3	(9.3,35.6)	20.1	(12.4,30.2)	**11.0**	(6.4,16.3)	**6.5**	(3.7,9.7)	**5.1**	(2.9,7.7)	**4.8**	(2.1,8.3)
Japan	World	7.4	(0.0,20.3)	**1.5**	(0.0,4.6)	**0.5**	(0.0,2.1)	**0.2**	(0.0,0.9)	**0.2**	(0.0,1.2)	**0.4**	(0.0,2.7)
	Country	13.8	(7.5,21.2)	10.8	(6.8,15.2)	11.3	(7.4,15.5)	12.9	(9.1,17.3)	11.3	(7.8,15.5)	11.0	(6.4,16.3)
	Total	22.3	(12.6,34.9)	**12.8**	(8.0,18.0)	**12.1**	(8.0,16.6)	**13.3**	(9.3,17.8)	**11.8**	(8.0,16.0)	**11.9**	(7.1,17.8)
Canada	World	11.3	(1.4,24.8)	8.5	(2.7,15.9)	7.2	(2.7,13.0)	5.3	(2.1,9.4)	**3.7**	(1.1,6.9)	**2.6**	(0.0,6.1)
	Country	9.8	(5.4,15.0)	7.3	(4.5,10.3)	**5.8**	(3.7,8.2)	**4.0**	(2.5,5.6)	**3.0**	(1.8,4.2)	**2.3**	(1.2,3.6)
	Total	21.8	(11.6,35.4)	16.0	(9.8,23.4)	**13.2**	(8.2,18.8)	**9.3**	(5.8,13.5)	**6.7**	(4.0,10.3)	**5.1**	(2.0,9.0)
Australia	World	2.6	(0.0,8.4)	3.2	(0.0,7.1)	3.8	(0.7,7.9)	2.7	(0.6,5.5)	1.6	(0.0,3.6)	0.6	(0.0,2.7)
	Country	19.6	(11.7,28.4)	18.5	(12.9,24.4)	**12.7**	(8.6,17.2)	**8.5**	(5.6,11.7)	**6.8**	(4.4,9.4)	**5.6**	(3.1,8.4)
	Total	23.1	(13.8,34.2)	22.1	(15.7,29.7)	16.8	(11.6,22.5)	**11.5**	(7.9,15.8)	**8.6**	(5.5,11.9)	**6.6**	(3.5,10.2)
New Zealand	World	4.0	(0.0,14.9)	3.0	(0.0,9.1)	2.5	(0.0,6.9)	2.1	(0.0,5.5)	1.6	(0.0,4.6)	1.2	(0.0,4.7)
	Country	204	(124,292)	151	(105,206)	**108**	(77,143)	**58.8**	(39.0,81.0)	**35.3**	(22.8,50.9)	**21.7**	(10.7,34.4)
	Total	210	(129,298)	155	(110,211)	**111**	(80,147)	**61.4**	(42.4,84.8)	**37.5**	(24.2,52.8)	**23.6**	(12.1,37.4)
U.K.	World	6.3	(0.0,16.8)	3.5	(0.0,8.1)	2.5	(0.1,5.4)	2.7	(0.7,5.2)	1.7	(0.2,3.5)	0.8	(0.0,2.6)
	Europe	0.7	(0.0,5.1)	0.8	(0.0,3.9)	0.5	(0.0,2.2)	0.1	(0.0,0.6)	0.1	(0.0,0.6)	0.1	(0.0,0.9)
	Country	16.5	(9.6,24.7)	14.9	(10.0,20.1)	**7.9**	(5.3,10.8)	**4.3**	(2.8,6.0)	**2.5**	(1.6,3.6)	**1.7**	(0.9,2.7)
	Total	25.9	(15.7,38.5)	20.5	(14.2,27.3)	**11.5**	(7.4,16.0)	**7.4**	(4.7,10.4)	**4.5**	(2.5,6.7)	**2.9**	(1.2,5.2)
Austria	World	1.5	(0.0,8.2)	0.4	(0.0,2.3)	0.2	(0.0,0.9)	0.1	(0.0,0.5)	0.1	(0.0,0.6)	0.4	(0.0,1.9)
	Europe	8.3	(1.3,16.8)	5.3	(1.3,10.3)	4.7	(0.9,9.2)	**2.8**	(0.7,5.4)	**1.3**	(0.0,3.0)	**1.1**	(0.0,3.3)
	Country	17.5	(10.4,25.6)	**11.4**	(7.7,15.5)	**6.6**	(4.3,9.0)	**3.4**	(2.1,4.9)	**2.2**	(1.2,3.2)	**1.6**	(0.7,2.6)
	Total	29.5	(19.0,41.8)	**17.9**	(12.2,24.6)	**11.8**	(7.3,16.8)	**6.5**	(4.0,9.7)	**3.8**	(1.9,6.0)	**3.6**	(1.4,6.5)
Belgium	World	0.3	(0.0,2.3)	0.1	(0.0,0.5)	0.0	(0.0,0.2)	0.0	(0.0,0.3)	0.1	(0.0,0.4)	0.5	(0.0,1.6)
	Europe	2.4	(0.0,6.2)	1.1	(0.0,2.8)	**0.1**	(0.0,0.4)	**0.1**	(0.0,0.5)	**0.1**	(0.0,0.8)	0.4	(0.0,2.2)
	Country	3.2	(1.6,5.2)	2.7	(1.4,4.2)	2.2	(1.2,3.5)	2.1	(1.2,3.4)	2.3	(1.2,3.6)	2.2	(0.9,3.9)
	Total	6.7	(3.1,11.4)	4.2	(2.3,6.4)	**2.4**	(1.3,3.7)	**2.4**	(1.3,3.7)	**2.7**	(1.5,4.1)	3.7	(1.8,6.1)
Denmark	World	3.7	(0.0,9.8)	2.8	(0.0,6.6)	1.8	(0.0,4.5)	1.5	(0.0,3.8)	1.7	(0.0,4.2)	1.5	(0.0,4.8)
	Europe	0.8	(0.0,3.3)	1.8	(0.0,4.9)	1.0	(0.0,3.5)	0.6	(0.0,2.6)	0.5	(0.0,2.3)	0.5	(0.0,2.8)
	Country	8.0	(4.0,12.4)	13.4	(9.1,18.3)	15.9	(11.0,21.2)	17.5	(12.5,22.8)	12.8	(8.8,17.6)	9.7	(5.5,14.6)
	Total	13.6	(6.8,21.8)	18.9	(12.9,25.4)	19.5	(13.7,26.4)	20.3	(14.5,26.9)	15.7	(11.0,21.6)	12.7	(7.4,19.5)
Finland	World	0.7	(0.0,4.7)	0.4	(0.0,2.4)	1.1	(0.0,4.0)	3.2	(0.0,8.1)	3.2	(0.0,8.3)	3.0	(0.0,9.2)
	Europe	0.5	(0.0,2.9)	0.3	(0.0,1.6)	0.2	(0.0,1.4)	0.2	(0.0,1.4)	0.3	(0.0,1.7)	0.4	(0.0,2.8)
	Country	13.0	(7.4,19.6)	15.5	(10.2,21.5)	13.5	(8.5,18.8)	13.1	(8.4,18.7)	11.5	(7.3,16.6)	10.6	(5.6,16.8)
	Total	15.7	(9.1,23.9)	16.9	(10.9,23.3)	15.7	(10.1,21.8)	17.5	(11.9,23.8)	16.1	(10.7,22.4)	15.6	(8.4,23.9)
France	World	2.4	(0.0,7.5)	0.6	(0.0,1.8)	**0.1**	(0.0,0.8)	0.6	(0.0,1.8)	0.3	(0.0,1.1)	0.5	(0.0,2.1)
	Europe	3.8	(0.2,8.0)	1.3	(0.0,2.9)	1.8	(0.0,4.0)	1.7	(0.2,3.6)	2.3	(0.2,4.9)	0.9	(0.0,3.0)
	Country	2.4	(1.4,3.6)	2.3	(1.5,3.2)	2.2	(1.4,3.1)	2.1	(1.4,3.0)	2.0	(1.3,2.9)	2.2	(1.2,3.3)
	Total	9.5	(4.6,15.3)	**4.4**	(2.5,6.6)	**4.4**	(2.3,7.0)	**4.7**	(2.6,7.1)	**4.9**	(2.5,7.7)	**4.0**	(1.8,7.1)
Germany	World	4.3	(0.0,13.4)	2.7	(0.0,6.3)	1.3	(0.0,3.5)	**0.3**	(0.0,1.4)	**0.2**	(0.0,1.2)	**0.3**	(0.0,1.5)
	Europe	4.3	(0.0,11.4)	3.3	(0.3,7.1)	4.3	(0.7,8.7)	6.4	(2.3,11.6)	3.5	(0.5,7.0)	2.6	(0.0,6.3)
	Country	8.1	(3.8,12.7)	8.0	(4.8,11.3)	7.9	(5.0,11.0)	7.6	(4.8,10.4)	5.8	(3.8,8.2)	**3.5**	(1.8,5.6)
	Total	18.7	(11.4,28.0)	14.7	(9.8,20.2)	14.1	(9.3,19.9)	14.7	(9.6,20.5)	**9.9**	(6.1,14.3)	**6.9**	(3.1,11.5)
Ireland	World	0.2	(0.0,1.4)	0.1	(0.0,0.7)	0.3	(0.0,1.5)	0.4	(0.0,2.1)	0.5	(0.0,2.6)	0.5	(0.0,3.4)
	Europe	0.8	(0.0,3.4)	0.7	(0.0,2.4)	0.3	(0.0,1.9)	0.2	(0.0,1.3)	0.5	(0.0,2.7)	1.0	(0.0,7.1)
	Country	4.3	(1.8,8.2)	6.2	(2.9,11.5)	9.2	(4.6,17.1)	15.1	(7.1,27.9)	30.5	(14.2,57.5)	94.4	(37.3,189)
	Total	6.1	(2.5,11.6)	7.5	(3.6,13.2)	10.5	(5.2,18.4)	16.5	(8.0,29.1)	32.7	(15.6,59.5)	98.5	(40.5,193)
Italy	World	0.7	(0.0,4.8)	0.8	(0.0,3.8)	1.0	(0.0,3.1)	0.8	(0.0,2.3)	0.4	(0.0,1.6)	0.2	(0.0,1.4)
	Europe	5.2	(0.0,11.9)	3.9	(0.0,8.0)	2.2	(0.0,5.3)	1.6	(0.0,3.9)	2.5	(0.0,5.7)	2.9	(0.0,7.5)
	Country	8.4	(4.9,12.1)	8.0	(5.5,10.9)	6.3	(4.2,8.6)	5.9	(4.0,8.0)	**5.2**	(3.4,7.0)	**3.3**	(1.7,5.1)
	Total	15.6	(8.4,24.3)	13.6	(8.7,19.0)	10.1	(6.3,14.8)	**8.8**	(5.6,12.5)	**8.5**	(5.4,12.4)	**7.0**	(3.2,12.1)
Sweden	World	0.8	(0.0,4.7)	1.1	(0.0,3.9)	1.9	(0.0,4.7)	2.8	(0.3,5.8)	2.9	(0.5,6.3)	2.6	(0.0,6.2)
	Europe	0.7	(0.0,3.7)	0.9	(0.0,3.4)	1.2	(0.0,3.9)	1.6	(0.0,4.1)	1.7	(0.0,4.5)	0.3	(0.0,1.6)
	Country	23.4	(13.6,34.5)	26.0	(18.7,34.7)	16.2	(11.1,21.8)	**9.7**	(6.3,13.4)	**5.5**	(3.3,8.0)	**2.8**	(1.3,4.7)
	Total	26.5	(15.8,38.5)	29.1	(21.2,38.6)	20.1	(13.7,26.9)	**14.8**	(9.8,20.0)	**10.8**	(6.7,15.6)	**6.2**	(2.6,10.8)
Netherlands	World	0.9	(0.0,4.4)	2.5	(0.0,6.1)	1.7	(0.0,4.1)	0.7	(0.0,2.1)	1.0	(0.0,2.5)	0.8	(0.0,3.1)
	Europe	2.7	(0.0,6.6)	3.6	(0.4,7.8)	4.1	(0.5,8.5)	3.3	(0.7,6.6)	1.6	(0.0,3.8)	4.8	(0.6,10.6)
	Country	14.3	(7.6,21.9)	17.5	(11.8,24.0)	12.9	(8.7,17.3)	**8.2**	(5.5,11.6)	**5.5**	(3.4,7.9)	**3.3**	(1.5,5.4)
	Total	19.2	(10.6,28.3)	24.6	(17.6,32.5)	19.4	(13.3,26.2)	12.8	(8.6,17.8)	**8.5**	(5.1,12.4)	**9.6**	(4.6,16.1)
Norway	World	2.1	(0.0,8.9)	1.7	(0.0,5.2)	0.9	(0.0,2.9)	0.3	(0.0,1.4)	0.3	(0.0,1.4)	0.3	(0.0,1.9)
	Europe	1.2	(0.0,5.6)	0.6	(0.0,2.9)	0.3	(0.0,1.9)	0.2	(0.0,1.5)	0.6	(0.0,2.5)	1.8	(0.0,6.0)
	Country	79.4	(49.5,116)	**44.7**	(28.1,62.3)	**29.6**	(18.3,42.1)	**24.6**	(15.9,35.4)	**22.3**	(14.5,31.1)	**16.0**	(8.7,25.1)
	Total	85.3	(54.7,122)	**48.3**	(31.8,66.7)	**31.5**	(19.8,44.3)	**25.7**	(16.6,36.6)	**23.8**	(15.7,33.1)	**19.4**	(10.9,29.4)
Switzerland	World	0.4	(0.0,3.4)	0.6	(0.0,2.8)	0.7	(0.0,3.0)	0.7	(0.0,2.5)	0.5	(0.0,2.2)	0.8	(0.0,3.4)
	Europe	1.6	(0.0,5.7)	2.8	(0.0,6.6)	2.2	(0.0,5.8)	2.4	(0.0,6.0)	2.2	(0.0,5.8)	2.8	(0.0,8.1)
	Country	5.4	(3.0,8.3)	7.5	(4.9,10.9)	7.6	(4.6,11.2)	7.3	(4.3,10.9)	6.0	(3.4,9.1)	4.2	(1.9,7.0)
	Total	8.7	(4.1,14.5)	11.8	(7.4,17.4)	11.4	(6.7,17.0)	11.1	(6.4,16.5)	9.5	(5.4,14.7)	8.9	(4.1,15.5)
Spain	World	0.7	(0.0,5.2)	0.1	(0.0,0.9)	0.2	(0.0,1.2)	0.2	(0.0,1.4)	0.2	(0.0,1.3)	0.2	(0.0,1.3)
	Europe	0.8	(0.0,3.8)	0.8	(0.0,2.7)	1.3	(0.0,3.9)	1.7	(0.0,5.0)	1.3	(0.0,4.1)	0.3	(0.0,2.0)
	Country	4.3	(2.1,6.6)	5.6	(3.3,8.0)	8.5	(5.1,12.2)	12.3	(7.5,18.1)	9.0	(5.1,13.6)	5.5	(2.5,9.5)
	Total	7.0	(3.2,12.5)	7.1	(4.1,10.3)	10.6	(6.5,15.5)	15.0	(9.5,21.6)	11.2	(6.7,16.8)	6.7	(3.0,11.6)

Notes: See next page

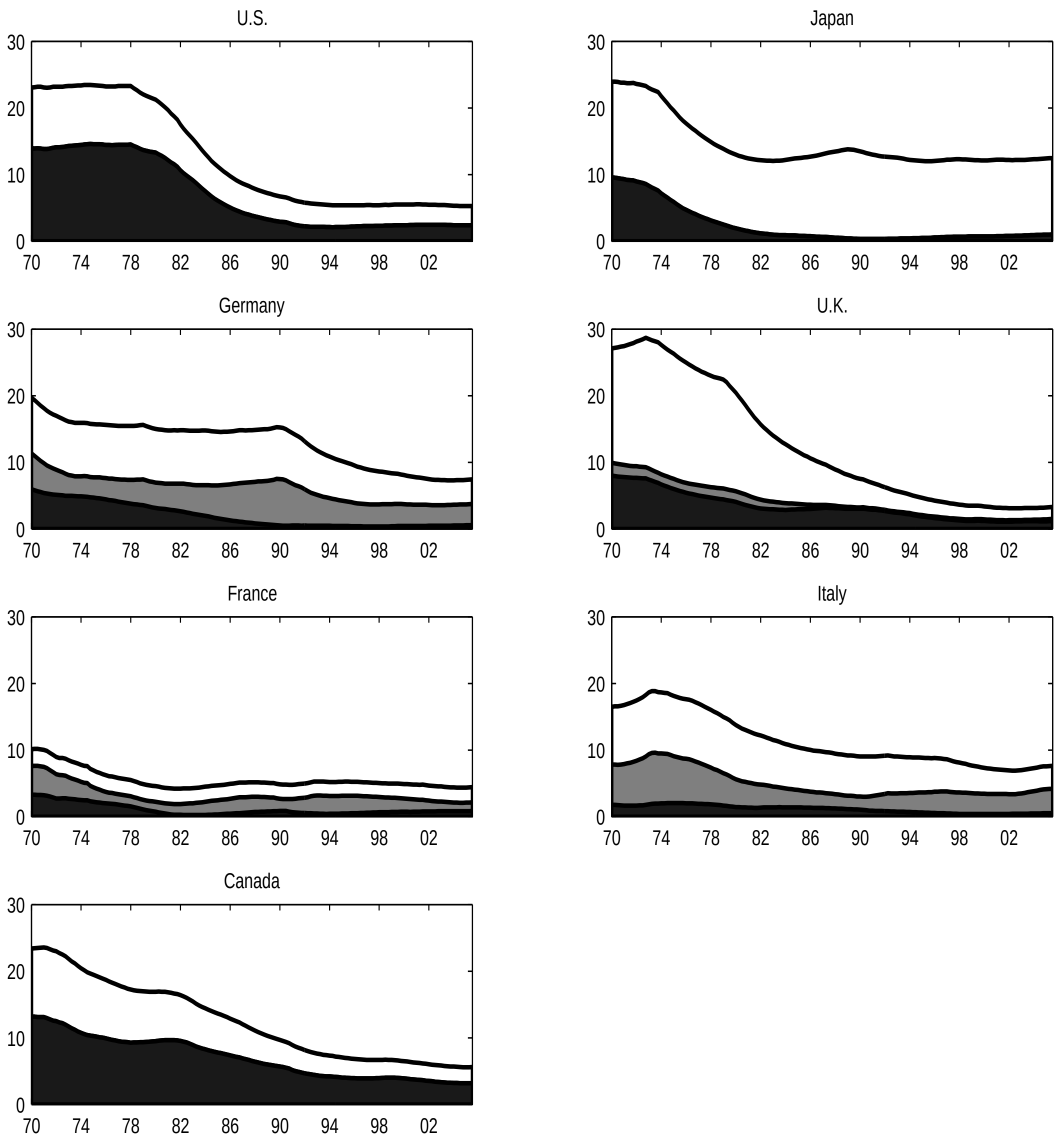
Notes to Table 1: The Table shows the variance attributable to international business cycles (World), European business cycles (Europe), country-specific fluctuations (Country), and the overall variance (Total), computed as the sum of the variances attributed to each component. For each date, each country, and each MCMC draw the variance attributable to international (European) business cycles is computed by multiplying ${\beta^w_{i,t}}^2$ (${\beta^e_{i,t}}^2$) times the variance of the factor, computed from equation (6) using the time t estimate of the factor's stochastic volatility ($h_{0,t}$). Likewise, the variance attributable to country-specific cycles is computed from equation (7) using the time t estimate of the idiosyncratic component's stochastic volatility ($h_{i,t}$). The Table shows the median across all posterior draws. Bold-faced numbers indicate that the variance has significantly declined (at the 10% level) relative to the beginning of the sample. Underlined numbers indicate that the variance has significantly declined (at the 10% level) relative to the previous date.

Table 2: Relative Importance of World, European, and Idiosyncratic Shocks

Dates		70-Q2	80-Q1	85-Q1	90-Q1	95-Q1	05-Q4
U.S.	World	58 (22,88)	61 (34,88)	54 (30,76)	42 (19,64)	36 (12,61)	40 (0,66)
	Country	42 (12,78)	39 (12,66)	46 (24,70)	58 (36,81)	64 (39,88)	60 (34,100)
Japan	World	35 (0,65)	12 (0,32)	**4** (0,16)	**1** (0, 7)	**2** (0,10)	3 (0,20)
	Country	65 (35,100)	88 (68,100)	**96** (84,100)	**99** (93,100)	**98** (90,100)	97 (80,100)
Canada	World	53 (21,80)	54 (32,76)	56 (33,75)	57 (37,77)	55 (33,77)	53 (18,81)
	Country	47 (20,79)	46 (24,68)	44 (25,67)	43 (23,63)	45 (23,67)	47 (19,82)
Australia	World	12 (0,31)	14 (0,29)	23 (6,42)	24 (7,42)	19 (2,38)	10 (0,33)
	Country	88 (69,100)	86 (71,100)	77 (58,94)	76 (58,93)	81 (62,98)	90 (67,100)
New Zealand	World	2 (0, 7)	2 (0, 6)	2 (0, 6)	3 (0, 9)	4 (0,12)	5 (0,18)
	Country	98 (93,100)	98 (94,100)	98 (94,100)	97 (91,100)	96 (88,100)	95 (82,100)
U.K.	World	26 (0,53)	17 (0,35)	22 (4,42)	37 (16,59)	38 (14,63)	28 (0,59)
	Europe	3 (0,20)	4 (0,19)	4 (0,18)	1 (0, 8)	2 (0,12)	5 (0,27)
	Country	66 (39,92)	75 (54,93)	70 (49,91)	60 (38,80)	58 (34,82)	60 (30,96)
Austria	World	5 (0,26)	2 (0,12)	1 (0, 8)	1 (0, 7)	3 (0,16)	12 (0,45)
	Europe	29 (7,51)	30 (11,50)	40 (17,62)	43 (21,65)	34 (5,62)	33 (0,64)
	Country	61 (40,83)	65 (46,83)	57 (35,81)	54 (33,77)	59 (32,86)	46 (17,80)
Belgium	World	4 (0,32)	1 (0,12)	1 (0, 9)	2 (0,11)	2 (0,15)	14 (0,41)
	Europe	38 (0,66)	28 (0,53)	**2** (0,14)	**3** (0,18)	5 (0,27)	13 (0,48)
	Country	50 (19,82)	67 (39,93)	**94** (79,100)	**93** (75,100)	**89** (66,100)	64 (33,95)
Denmark	World	28 (0,55)	15 (0,31)	9 (0,21)	7 (0,17)	11 (0,25)	12 (0,33)
	Europe	6 (0,23)	10 (0,24)	5 (0,17)	3 (0,12)	3 (0,14)	4 (0,20)
	Country	61 (34,88)	72 (53,90)	84 (68,98)	**88** (75,99)	83 (67,98)	80 (58,100)
Finland	World	5 (0,26)	2 (0,13)	7 (0,23)	19 (0,41)	21 (0,44)	20 (0,48)
	Europe	3 (0,18)	1 (0, 9)	1 (0, 8)	1 (0, 8)	2 (0,10)	3 (0,16)
	Country	87 (65,100)	94 (82,100)	89 (73,100)	78 (56,99)	75 (51,98)	72 (46,100)
France	World	27 (0,61)	13 (0,36)	3 (0,16)	14 (0,32)	5 (0,21)	12 (0,41)
	Europe	43 (9,73)	29 (4,55)	42 (13,70)	37 (13,62)	48 (18,75)	23 (0,53)
	Country	26 (10,46)	**53** (29,77)	**51** (24,79)	46 (23,70)	42 (20,69)	**56** (26,90)
Germany	World	23 (0,58)	19 (0,38)	9 (0,23)	2 (0, 9)	2 (0,11)	4 (0,21)
	Europe	24 (0,57)	23 (5,45)	31 (11,54)	44 (24,64)	36 (13,59)	40 (9,69)
	Country	45 (17,71)	55 (34,76)	57 (35,80)	52 (32,73)	60 (37,82)	52 (24,81)
Ireland	World	3 (0,21)	1 (0, 8)	2 (0,14)	3 (0,12)	1 (0, 8)	0 (0, 4)
	Europe	13 (0,43)	9 (0,28)	3 (0,16)	1 (0, 8)	1 (0, 8)	1 (0, 7)
	Country	77 (49,100)	87 (68,100)	92 (75,100)	95 (82,100)	96 (86,100)	**98** (90,100)
Italy	World	4 (0,28)	6 (0,26)	10 (0,27)	9 (0,24)	5 (0,17)	3 (0,19)
	Europe	35 (0,58)	30 (0,49)	23 (0,43)	19 (0,37)	30 (6,56)	44 (11,76)
	Country	55 (30,81)	60 (39,82)	64 (41,87)	69 (47,91)	62 (38,87)	49 (20,81)
Sweden	World	3 (0,16)	4 (0,13)	10 (0,22)	**20** (5,37)	**28** (7,50)	**43** (11,75)
	Europe	3 (0,14)	3 (0,11)	6 (0,18)	11 (0,25)	16 (0,36)	4 (0,23)
	Country	91 (76,100)	91 (81,100)	82 (66,97)	**67** (47,86)	**52** (30,74)	**47** (19,78)
Netherlands	World	5 (0,21)	10 (0,23)	9 (0,20)	5 (0,15)	12 (0,27)	8 (0,30)
	Europe	14 (0,32)	15 (2,30)	21 (5,40)	27 (9,45)	19 (0,38)	**52** (21,80)
	Country	77 (57,95)	73 (55,88)	68 (49,86)	66 (46,85)	67 (45,88)	**35** (13,60)
Norway	World	3 (0,10)	3 (0,11)	3 (0, 9)	1 (0, 5)	1 (0, 6)	2 (0, 9)
	Europe	1 (0, 7)	1 (0, 6)	1 (0, 6)	1 (0, 6)	3 (0,10)	10 (0,28)
	Country	95 (86,100)	94 (86,100)	95 (87,100)	97 (91,100)	95 (86,100)	86 (67,100)
Switzerland	World	5 (0,34)	5 (0,23)	7 (0,25)	6 (0,21)	6 (0,21)	10 (0,35)
	Europe	20 (0,50)	25 (0,46)	20 (0,41)	22 (0,44)	24 (0,48)	34 (0,64)
	Country	65 (37,97)	66 (43,89)	69 (45,93)	68 (44,92)	66 (40,91)	49 (21,84)
Spain	World	10 (0,55)	2 (0,13)	2 (0,11)	2 (0, 9)	2 (0,11)	3 (0,18)
	Europe	12 (0,43)	12 (0,32)	12 (0,31)	12 (0,30)	12 (0,32)	5 (0,25)
	Country	66 (33,100)	82 (63,100)	83 (64,100)	85 (66,100)	84 (63,100)	88 (65,100)

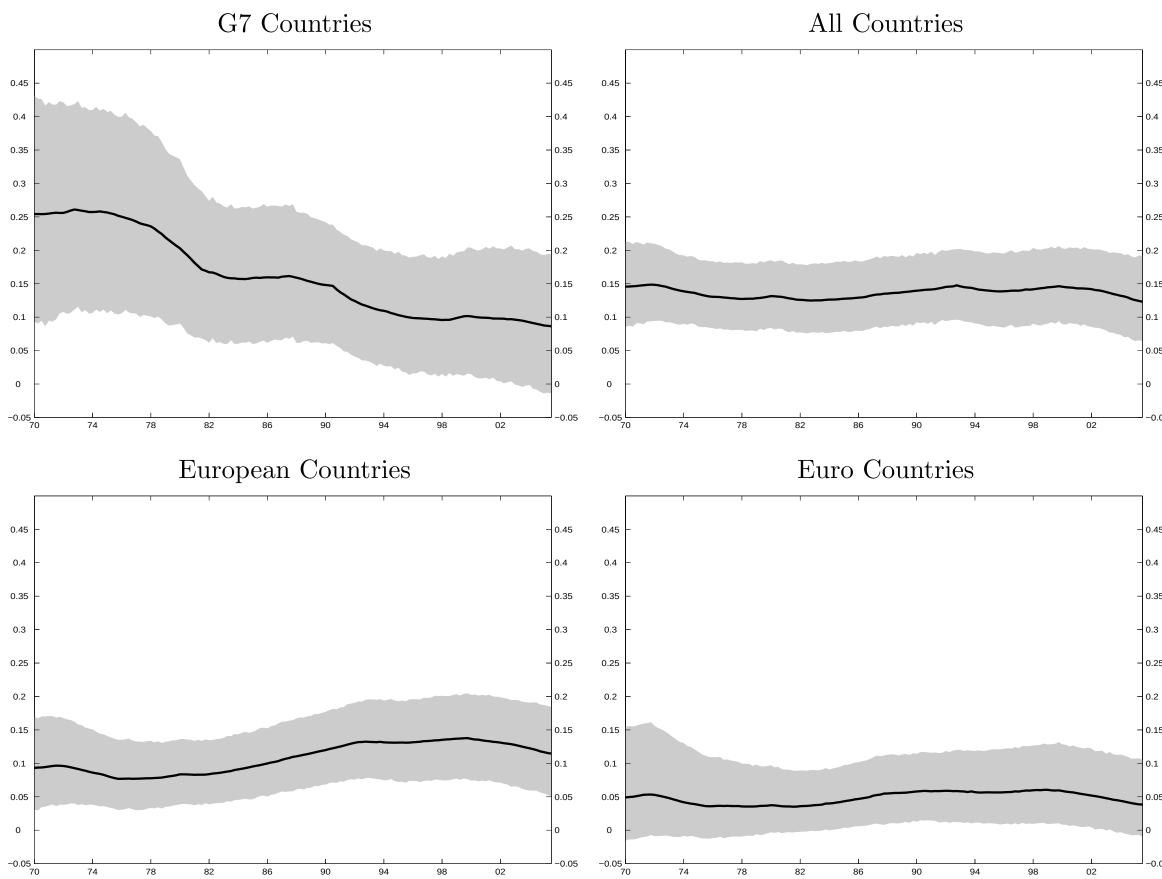
Notes: The Table shows the variance attributable to world, European, and country-specific cycles as a fraction of the total variance for each country in the sample for the six dates shown above. All figures are in percent, and numbers in parenthesis represent the 90% posterior bands. Bold-faced figures indicate that the change in the fraction of the variance explained by each component relative to the beginning of the sample is significant at the 10% level. Underlined figures indicate that the change relative to the previous date is significant at the 10% level.

Figure 1: Volatility: Common Factors vs Country-Specific Fluctuations (G7 Countries)



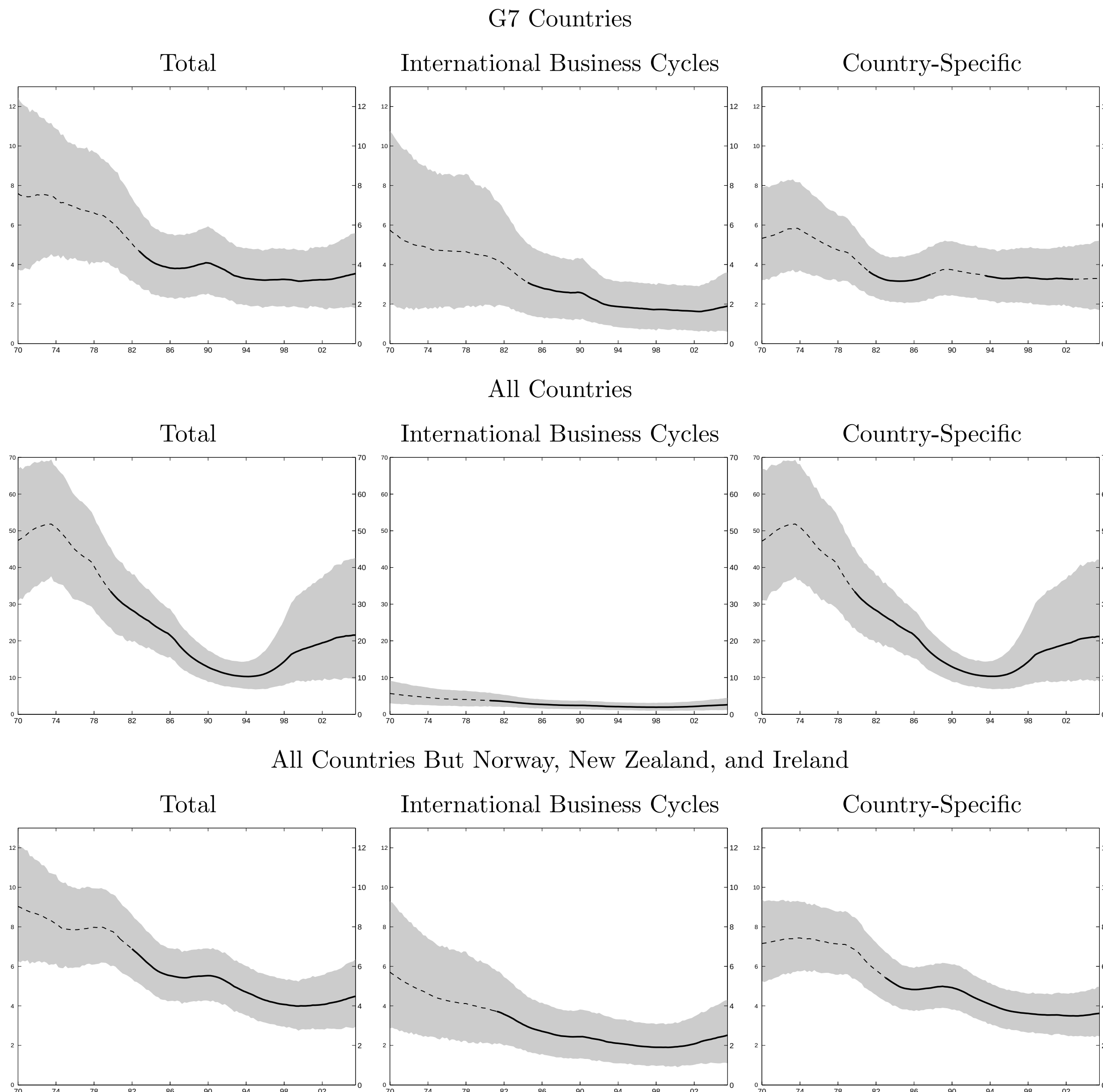
Notes: The Figure shows the variance attributable to international business cycles (black), European business cycles (gray), and country-specific fluctuations (white). For each point in time, each country, and each MCMC draw the variance attributable to international (European) business cycles is computed by multiplying ${\beta^{w}_{i,t}}^2$ (${\beta^{e}_{i,t}}^2$) times the variance of the factor, computed from equation (6) using the time t estimate of the factor's stochastic volatility ($h_{0,t}$). Likewise, the variance attributable to country-specific cycles is computed from equation (7) using the time t estimate of the idiosyncratic component's stochastic volatility ($h_{i,t}$). The Figure shows the median across all posterior draws.

Figure 2: Average Cross-country Correlations



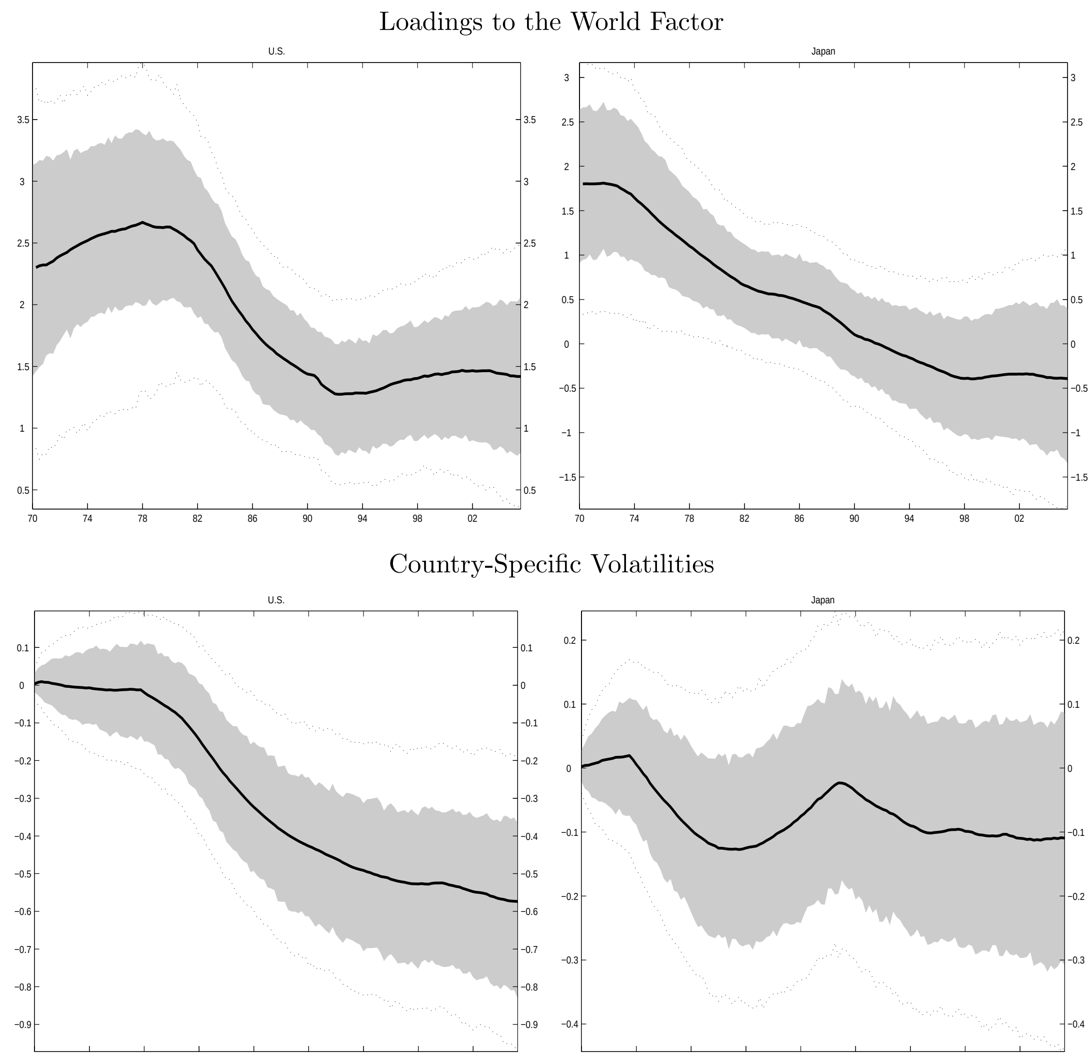
Notes: The figure shows the average cross-country correlation for G7 countries, all countries in the sample, European countries, and countries that have joined the Euro. At each point in time, and for each MCMC draw, we compute all pair-wise correlations implied by the factor model using the time t estimates of the loadings and stochastic volatilities. We then take the (unweighted) average across countries in the group. The Figure plots the median and the 90% bands of this average.

Figure 3: Cross-sectional Dispersion in Volatility



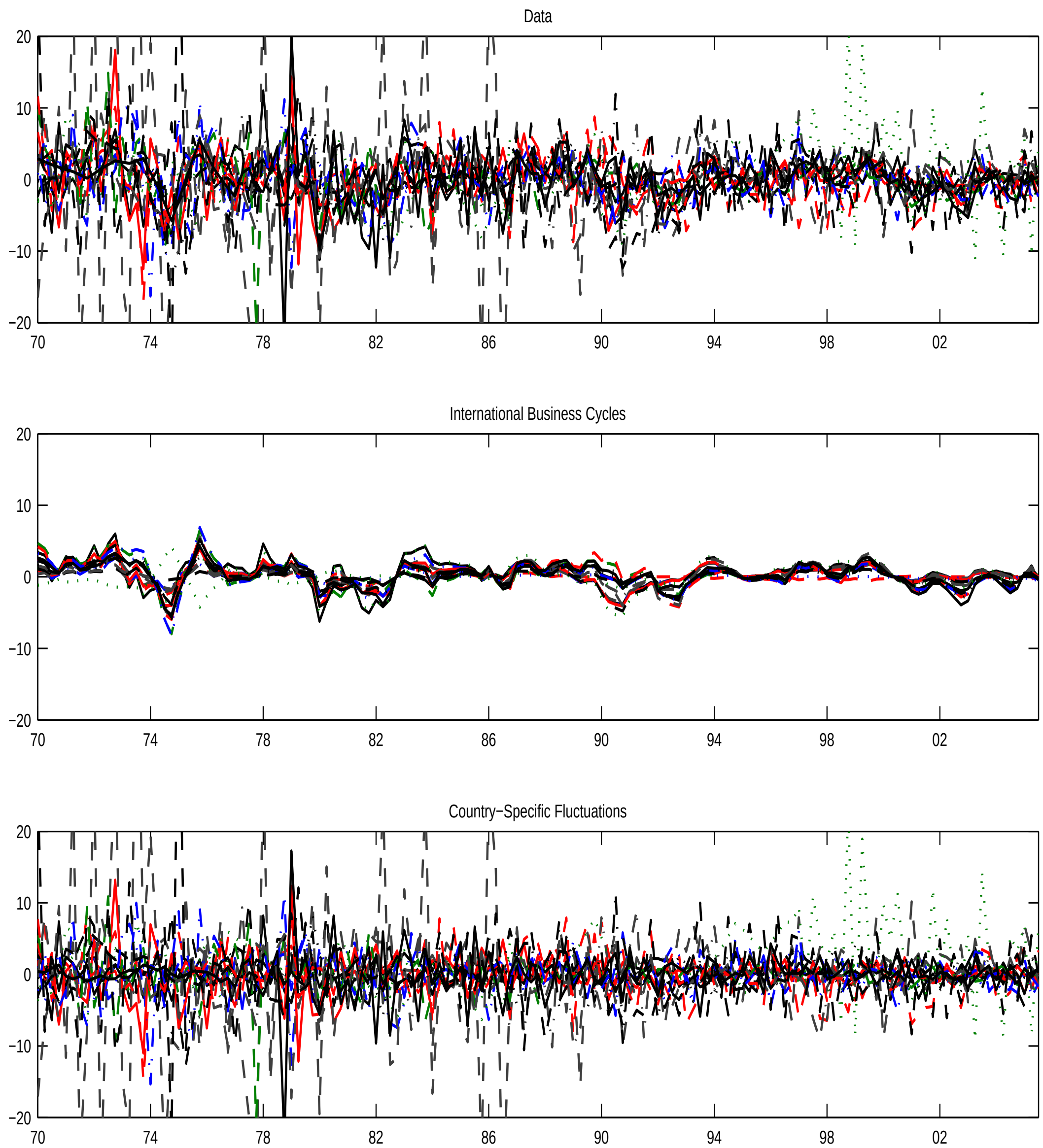
Notes: The Figure shows the evolution over time of the cross-sectional standard deviation in output growth volatility for three groups of countries: G7, all the countries in our sample, and all the countries but Norway, New Zealand, and Ireland. For each group of countries we show the median across MCMC draws of cross-sectional standard deviation of the total volatility, as well as the volatility attributed to international business cycles and country-specific fluctuations. The shaded area represents the 90% bands. The median is shown as a solid line whenever the decline in the cross-sectional standard deviation relative to the beginning of the sample is significant at the 10% level, and as a dashed line otherwise.

Figure 4: The US and Japan: Sensitivity to the World Factor and Country-Specific Volatilities



Notes: The Figure plots the time-varying loadings to the world factor $b^w_{i,t}$ (top) and standard deviations of the country-specific component relative to the first period $\sigma_{i,t}/\sigma_{i,1}$ (bottom) for the US and Japan. The solid lines show the median and the shaded areas and dotted lines represent the 68 and 90% bands, respectively.

Figure 5: Output Growth Decomposition: International Business Cycles vs Country-Specific Fluctuations



Notes: The Figure plots for each country the demeaned data (Data), the posterior medians of the terms $b^w_{i,t} f^w_t + b^e_{i,t} f^e_t$ (International Business Cycles) and $\epsilon_{i,t}$ (Country-Specific Fluctuations) in equation (9).