

## Has mathematics nothing to do with organized thought?

How many spoons are there in a cup? What is the thickness of a hair (in inches)? The ancient Romans used X for 10, V for 5, II for 2, etc. Try to be a Roman and multiply XII by IV. Our conventional rules learned at a tender age cannot be applied any longer. Why?

Your belt is one yard two inches and  $1/4$  long. You went on a diet and your waist line has shrunk by 3 inches. How long does your belt need to be now? Our system of paces, feet and knuckles is as easy to deal with as the Romans', wouldn't you agree?

Some argue that the dismal state of mathematical awareness in our school system is just a reflection of the society at large. This society for whom the intellectual values are far secondary to those measured in \$\$\$. Thus, they say, we prepare students in very narrow "applicable" fields, and do it increasingly badly (using simple-minded drills) because the breadth of knowledge is lacking.

I don't disagree, in general. But in particular, I want to argue that part of our problems in teaching/learning mathematics is a result of the system in which we count and measure. A system that is cumbersome to handle and literally forces one to use stupefying drills instead of gaining some insight from the problems we solve.

*The metric system was adopted in the revolutionary France in 1799 replacing the confusing welter of traditional but illogical units of a measure with a rational system based on a natural unit, meter,  $m = 10^{-7}$  of the length of a quadrant of the meridian. The name meter is derived from metron, measure.*

That much dictionary. Why are the traditional units illogical? They all come from our environment (e.g., foot) and moreover we seem to like tradition. And why is the new, metric system rational? Is the meridian better than our own waist line?

The beauty and simplicity of the metric system is that it is based on the *positional system* of writing numerals. When Romans wrote a larger 'digit' before a smaller one, i.e. VI, they meant: add the two, i.e.  $VI = V + I (= 6)$ . When they wrote a larger 'digit' after a smaller one, i.e. IV, they meant: subtract the smaller from the larger one, i.e.  $IV = V - I (= 4)$ . When we write 51 we mean  $50 + 1$ , i.e. digits have different values depending where they are *positioned*. The rightmost digit has its 'face' value, while the penultimate digit is multiplied by ten, i.e.  $50 = 5 \times 10$ , etc. Similarly, numeral 1994 means  $1000 + 900 + 90 + 4$ , or  $1 \times 10^3 + 9 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$ , or

<i>position</i>	3	2	1	0
<i>digit</i>	1	9	9	4

Thus, when we want to add (or subtract) two numerals, we place them properly positioned and perform the operations we learned it at school, from right to left, from less significant digit to more significant one.

These were integer numerals. How to generalize this idea to non-integer ones? We simply extend our sequence of indices denoting the position to nega-

tive ones, placing a 'dot' after the '0' position, i.e.  $53\frac{1}{4}$  (fifty three and a quarter) will be represented as 53.25, or  $5 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$ , or

<i>position</i>	1	0	-1	-2	
<i>digit</i>	5	3	2	5	.

As a consequence, we never write numerals in a cumbersome form  $53\frac{1}{4}$  (Again, subtract two numerals given in yards, feet and inches and you will get a headache. Note that there are 12 inches in a foot, 3 feet in a yard, etc., i.e. the multiplicity of each consecutive measure is different). Instead, we write them in a positional form, 53.25. For example, 5 meters and a quarter minus 52 millimeters makes

$$\begin{array}{r}
 5.250 \\
 - \quad 0.052 \\
 \hline
 \text{equals } 5.198
 \end{array}
 \qquad \text{Simple, my Watson!}$$

Please note that in a metric system each measure is a power of ten of the basic unit, the meter. Thus,

$$\begin{aligned}
 \text{kilometer} &= \text{km} = 1000 \text{ m} = 10^3 \text{ m}, \\
 \text{centimeter} &= \text{cm} = \frac{1}{100} \text{ m} = 10^{-2} \text{ m}, \\
 \text{millimeter} &= \text{mm} = \frac{1}{1000} \text{ m} = 10^{-3} \text{ m}, \\
 \text{micron} &= \mu = \frac{1}{1,000,000} \text{ m} = 10^{-6} \text{ m}, \text{ etc.}
 \end{aligned}$$

We shall call this integer, 10, the *basis* of the metric system, and the system itself - decimal system.

The same applies to two and three-dimensional measures:

$$\begin{aligned}
 \text{square kilometer} &= \text{km}^2 = 1000^2 \text{ m}^2 = 10^6 \text{ m}^2, \\
 \text{square centimeter} &= \text{cm}^2 = \frac{1}{100^2} \text{ m}^2 = 10^{-4} \text{ m}^2, \\
 \text{cubic kilometer} &= \text{km}^3 = 1000^3 \text{ m}^3 = 10^9 \text{ m}^3, \\
 \text{cubic centimeter} &= \text{cm}^3 = \frac{1}{100^3} \text{ m}^3 = 10^{-6} \text{ m}^3, \text{ etc.}
 \end{aligned}$$

Let's see a similar situation in the traditional system:

$$\begin{aligned}
 12 \text{ inches} &= 1 \text{ foot}, \\
 3 \text{ feet} &= 1 \text{ yard}, \\
 5\frac{1}{2} \text{ yards} &= 1 \text{ rod}, \\
 40 \text{ rods} &= 1 \text{ furlong}, \\
 8 \text{ furlongs} &= 1 \text{ mile}.
 \end{aligned}$$

Even posed with a simple question: how many cubic inches there are in a cubic yard (Answer:  $3^3 \times 12^3 = 15, 552$ ) you would answer: give me a break. In the metric system all is simple:  $1\text{m}^3 = (100\text{cm})^3 = (10^2)^3\text{cm}^3 = 10^6\text{cm}^3$ .

The unit of mass, gram (g) was established as the mass of cubic centimeter. Thus the density of water (under normal conditions<sup>1</sup>) is  $1 \frac{\text{g}}{\text{cm}^3} = 1 \frac{\text{kg}}{\text{dm}^3} = 1 \frac{\text{t}}{\text{m}^3}$ .

An example.

A Martian with no earthly experience would like to estimate whether two Earthling kids, each at his end of the Pond, would be able to carry a bucket filled one with 12 liters of water, the other with 3 gallons (about the same, within 5% of error).

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<sup>1</sup>normal conditions: 4°C and atmospheric pressure at the sea level

The American kid might remember that the density of water is 62.43 pounds per cubic foot. Now, to find how many cubic feet there are in a gallon he needs to look it up in an encyclopedia (and smaller encyclopedias do not provide this information): 1 gallon = 231 cu. in. Now he needs to convert cubic feet into cubic inches:  $1ft^3 = 12^3in^3 = 1,728in^3$ . Then, he needs to multiply 3 gallons by 231 to get cubic inches and change to the required units (cubic feet) by dividing by 1,728. Finally, multiplying by 62.43 he gets 25.04 pounds. A lot of tedious, unnecessary work that does not give any insight, just makes you to hate the process of finding the answer.

The European kid answers immediately: 12 kilograms. How did she arrive at this result? She knows that the density of water is 1 kg per cubic decimeter, dm (dm = 0.1 m). And the liter is defined as the volume equivalent to  $1dm^3$ .

In calculations one can partition numbers into a mantissa and exponential parts (similar to the way computers operate).

An example.

Forty million, or 40,000,000, can be written as  $4 \times 10^7$ . To represent a number we only need the mantissa, 4, and the exponent, 7. Similarly, 31,234,567 can be *approximated* by  $3.1 \times 10^7$ .

Using this idea and the metric system one can perform complicated computations (approximately, within reasonable accuracy) with an amazing simplicity of means.

An example.

The US occupies how many percent of the land on the Earth?

Let's solve it in several stages.

1. First, compute an approximate value of the surface of the Earth (in  $km^2$ ). Knowing the definition of the meter and assuming that the Earth is approximately is a sphere of radius R, we get:  $2\pi R = 4 \times 10^7m$ . Thus,  $\pi R = 2 \times 10^7m$  and  $R = \frac{20}{\pi} \times 10^6m \approx 6 \times 10^6m$ .

Finally, the surface of the Earth is about  $S = 4\pi R^2 = 2\pi R \times 2R \approx 4 \times 10^7 \times 12 \times 10^6m^2 \approx 50 \times 10^{13}m^2 = 5 \times 10^8km^2$ .

2. Then, knowing that oceans cover 71% of the Earth surface and the surface of the US approximately is 9.4 millions square kilometers or  $9.4 \times 10^6km^2$  we get our answer

US occupies about  $\frac{S_{US}}{S_{Earthland}} \times 100\% \approx \frac{9.4 \times 10^6km^2}{0.29 \times 5 \times 10^8km^2} \times 10^2 \approx 6.5\%$  of the land on Earth.

Conclusion

Employing a rational metric system and properly organizing computations one can easily obtain sufficiently accurate approximations to many real life problems.