

Final Exam

Name: _____
(Print) (Last, First)

1	2	3	4	5	6	7	8	9	Total
/30	/30	/50	/80	/40	/30	/50	/50	/40	/400

Write clearly. Show work. Justify your answers.

1. Find the volume of the solid obtained by rotating $f(x) = \sin(x)$ about x-axis from 0 to π .

2. The linear density in a rod 8 m long is $\frac{12}{\sqrt{x+1}}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

3. Find the following limits, if they exist:

a. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$.

b. $\lim_{x \rightarrow 0} \frac{\sin(x)}{|x|}$.

c. $\lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{x}}$.

4. Evaluate the following integrals.

a. $\int x \ln x dx$

b. $\int \sin \sqrt{x} dx$

c. $\int \frac{x+5}{x^2+x-2} dx$

d. $\int \frac{2x+3}{(x+1)^2} dx$

5. Evaluate the following integrals, if they exist:

a. $\int_0^{\infty} e^{-x} dx$

b. $\int_{-1}^1 \frac{dx}{\sqrt[3]{x^2}}$

c. $\int_{-1}^1 \frac{dx}{x^2}$

6. Find the linearization of $f(x) = \sqrt[3]{1+3x}$ at $a = 0$.

7. Let a function be defined by its series expansion $f(x) = \sum_{n=0}^{\infty} \left(\frac{x-1}{2}\right)^n$.

a. Find the interval of convergence for $f(x)$.

b. What function has this series expansion?
(Hint: note its similarity with the geometric series).

8. a. Expand $\sin^2 t$ in powers of t up to t^6 .

b. Find the approximate value of $\int_0^1 \sin^2 t dt$ using the expansion from part a.

9. Evaluate $\int \frac{e^x}{x} dx$ as an infinite series.