

Exam #1

Name: _____
(Print) (Last, First)

1	2	3	4	5	Total
/40	/40	/40	/40	/40	/200

Write clearly. Show work. Justify your answers.

1. Consider the differential equation $y' + 2ty = t$.
 - a. Find the general solution.
 - b. Find the solution with the initial condition $y(0) = 0$.
 - c. Sketch its graph.

2a. Consider the nonlinear equation $y' = y^2$. Perform sign analysis, i.e. find the equilibrium solutions (constant solutions $y(t) = \text{const}$ for all t). Sketch the behavior of solution curves for $t \geq 0$.

2b. Find the solution of the Initial Value Problem $y' = y^2$ with the initial condition $y(0) = -1$. Sketch its graph.

- 3.** Consider the system of ODEs $x' = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix} x$.
- Find its eigenvalues and corresponding eigenvectors (check that matrices $(A - \lambda I)$ are singular),
 - Find the general solution to this system of ODEs,

4. Consider the system of ODEs $x' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} x$.
- Find its eigenvalues and corresponding eigenvectors,
 - Find the general solution to this system of ODEs in the *real valued* form.
 - If $x(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, find the solution to this Initial Value Problem.

5. Consider the system of ODEs $x' = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} x$.

- a. Find its eigenvalues and corresponding eigenvectors,
- b. Find the solution vectors and verify that they are linearly independent,
- c. Write the general solution to this system of ODEs.