

Exam #2

Name: _____
(Print) (Last, First)

1	2	3	4	5	Total
/40	/40	/40	/40	/40	/200

Write clearly. Show work. Justify your answers.

1. Consider the damped unforced mechanical system described by the equation $my'' + \gamma y' + \kappa y = 0$, where m , γ and κ all are positive. Show that its general solution $y(t) = c_1 y_1(t) + c_2 y_2(t)$ decays to zero as time t approaches ∞ .

Hint: consider separately the 3 cases: under-damped, critically damped and over-damped systems.

2a. Find the general solution of $y'' + 4y = \sin 2t$.

2b. Consider $y'' + 2y' + 5y = te^{-t} \sin 2t$. Find the correct form of the particular solution $Y(t)$, *without* computing the coefficients.

3. Consider an undamped forced oscillator

$$my'' + ky = F\cos(\omega t) \quad \text{with } y(0) = 0 \text{ and } y'(0) = 0.$$

Let $m = 1$, $F = 20$, $\omega_0 = 10$, and $\omega = 11$, where $\omega \neq \omega_0 = \sqrt{\frac{k}{m}}$. Compute the solution of this Initial Value Problem. Rewrite the solution as a product of sines. Observe that when $\omega \approx \omega_0$ (as here, $11 \approx 10$) the solution exhibits two frequencies: the fast one and the slow one (producing the envelope). Sketch the graph of the solution (two periods of it).

- 4a.** Give the definition of the Laplace transform of a given function $f(t)$.
4b. Using this definition compute Laplace transform of the following function:

$$f(t) = \begin{cases} 0 & \text{for } t \in [0, 1), \\ 1 & \text{for } t \geq 1. \end{cases}$$

- 4c.** Find Inverse Laplace transform of the following function:

$$Y(s) = \frac{s^3 - 4s + 8}{s^2(s^2 + 4)}$$

5. Solve the following Initial Value Problem using Laplace transform

$$y'' + 2y' + y = 4e^{-t},$$

$$y(0) = 2, y'(0) = -1.$$