

### Quiz #1 – Math 502a

1. Define the central difference approximation of the derivative as

$$D_h(x) = \frac{f(x+h) - f(x-h)}{2h}.$$

Find the value of  $h$  in the form of  $O(\epsilon^p)$ , where  $\epsilon$  is the machine precision, for which the error

$$e_h = |D_h(x) - f'(x)|$$

reaches the minimum. Take into consideration both the discretization and round off errors. Assume  $f(x)$  to be a sufficiently smooth function.

2. Let  $A \in R^{n \times n}$ . Prove that if  $A$  is *positive definite* (PD) then its symmetric part,  $\frac{1}{2}(A + A^T)$ , also is PD.

3. Let  $B \in R^{n \times n}$  be a skew-symmetric matrix, i.e.  $B^T = -B$ . [Note that  $BB^T = B^T B$ ]. Denote  $A = (I + B)(I - B)^{-1}$ .

Show that  $AA^T = A^T A = I$ .