

Quiz #2 – Math 502a

Problem 1.

Find the least squares fit $y(x) = a + bx$ to the data given by $f(x) = \sqrt{x}$

(a) on $x = \{0, .25, 1\}$,

(b) on $x \in [0, 1]$,

(c) compute the appropriate norms of the residuals in both cases.

Problem 2.

Let $A \in R^{m \times n}$. Show that $A^T A$ is symmetric positive semidefinite and that the normal equations $A^T A x = A^T b$ are always consistent.

Problem 3. Let $Q_1 = (q_1, \dots, q_n) \in R^{n \times n}$ be a real orthogonal matrix. Determine a reflector $P = I - 2vv^T$, such that $Pq_1 = e_1 = (1, 0, \dots, 0)^T$, and show that $Q_2 = PQ_1$ has the form

$$Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & \hat{Q}_2 \end{bmatrix},$$

where $\hat{Q}_2 = (\hat{q}_2, \dots, \hat{q}_n) \in R^{(n-1) \times (n-1)}$ is a real orthogonal matrix.