

### Quiz #3 – Math 502a

**Problem 1.** Let  $A = \begin{pmatrix} 3 & 3 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 2.9 \end{pmatrix}$ .

- Explain the slow rate of convergence of the power method with  $A$ .
- Choose a suitable shift  $\sigma$  so that power method converges to the largest eigenvalue of  $A$ . Explain the improvement of the rate of convergence with  $A - \sigma I$ .
- Find the optimal shift  $\sigma_{opt}$  so that power method converges to the largest eigenvalue of  $A$ .

**Problem 2.** Show that a product (both HR and RH) of a right triangular matrix,  $R$  with a Hessenberg matrix,  $H$  is a Hessenberg matrix.

**Problem 3.** Prove that a matrix  $A \in C^{n \times n}$  is normal,  $A^H A = A A^H$ , if and only if  $A$  is unitary diagonalizable, that is, there exists a unitary matrix  $U \in C^{n \times n}$  such that  $U^H A U = D = \text{diag}(\lambda_1, \dots, \lambda_n)$ .