

### Quiz #4 – Math 502

**Problem 1** Formulate the main theorem for SOR iterations that allows to find the optimal value of the parameter,  $\omega$  and the corresponding spectral radius  $\rho(Q_{\omega_{opt}})$ .

**Problem 2** Consider the  $n \times n$  matrix  $A$  defined by its entries

$$a_{ij} = \begin{cases} 2, & i = j, \\ -1, & |i - j| = 1, \\ 0, & \text{otherwise} \end{cases}$$

Define the Jacobi iteration matrix,  $Q_J$ , compute its eigenvalues and the spectral radius,  $\rho(Q_J)$ . Then, for the optimal SOR iteration applied to a problem with  $A$  and using the result from **1** compute the optimal value of the parameter,  $\omega$  and the corresponding spectral radius  $\rho(Q_{\omega_{opt}})$ . Finally, compute the (approximate) dependence between the rate of convergence,  $R_{\infty}(Q_{\omega_{opt}})$  and  $n$ , as well as between the number of iterations,  $N$  and  $n$ .