

Nonlinear Equations Project – Math 502b

Consider the following nonlinear problem $f(\mathbf{x}) = 0$, where

$$f(\mathbf{x}) = \begin{pmatrix} x_1^5 + x_2^3 x_3^4 + 1 \\ x_1^2 x_2 x_3 \\ x_3^4 - 1 \end{pmatrix}.$$

a. Find (by hand) all the zeros of this system,

b. Compute the Jacobian, $J(\mathbf{x})$

(Note that the Jacobian is singular for $x_3 = 0$).

c. Consider two initial guesses

i) $\mathbf{x}_0 = (-.01, -.01, -.01)^T$ and

ii) $\mathbf{x}_0 = (.1, .1, .1)^T$.

d. Compute both $\|J(\mathbf{x}_0)^{-1}\|$.

Note that these Jacobians are almost singular although the initial guesses are ‘not far’ from a root, i.e. $\|\mathbf{x} - \mathbf{x}_0\|$ is ‘small’.

Write a code for the Newton method to solve this problem starting with the given initial conditions. Notice some difficulties with convergence. As a remedy implement a damped Newton modification using the Armijo-Goldstein criterion.

Present the results for both algorithm with a detailed discussion of their performance.