Omitted appendices of The Singularity of the Information Matrix of the Mixed Proportional Hazard Model

Appendix A Existence of the score if $\mathrm{E}(V)=\infty$.

We have

$$\mathrm{E}(e^U|S)=\frac{\mathrm{E}\left(e^{2U}e^{-Se^U}\right)}{\mathrm{E}\left(e^U e^{-Se^U}\right)}$$

Because $x^2e^{-sx}\leq\frac{4}{s^2}e^{-2}$ the numerator is bounded by $\frac{4}{S^2}e^{-2}$. Because the distribution of e^U is not degenerate in 0, there are $0<v_1<v_2$ with $\Pr(v_1<e^U\leq v_2)>0$. Hence the denominator is greater than $\min\{v_1e^{-Sv_1},v_2e^{-Sv_2}\}\Pr(v_1<e^U\leq v_2)>0$. Hence

$$\mathrm{E}(e^U|S)\leq\frac{\frac{4}{S^2}e^{-2}}{\min\{v_1e^{-Sv_1},v_2e^{-Sv_2}\}\Pr(v_1<e^U\leq v_2)}$$

Appendix B The information matrix for translated Weibull baseline hazard.

The score is evaluated at $\alpha=\alpha_0$ so that

$$a_{11}=\frac{1}{\alpha_0}\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)-\frac{1}{\alpha_0}\mathrm{E}_X\left[\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)\right] \tag{27}$$

$$\begin{aligned}a_{12}=&\frac{e^{\beta_0'X}}{\alpha_0S}\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)-\\&-\mathrm{E}_X\left[\frac{e^{\beta_0'X}}{\alpha_0S}\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)\right]-\\&-\frac{e^{\beta_0'X}\varepsilon^{\alpha_0}\ln\varepsilon}{S}+\mathrm{E}_X\left[\frac{e^{\beta_0'X}\varepsilon^{\alpha_0}\ln\varepsilon}{S}\right].\end{aligned} \tag{28}$$

To see that the distribution of the efficient score is nonsingular, consider the special case $U\equiv 0$ so that $\mathrm{E}\left[e^U|S\right]=1$. Then a necessary condition for singularity is that

$$\begin{aligned}a_{11}-a_{12}S=&\frac{1}{\alpha_0}\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)-\frac{1}{\alpha_0}\mathrm{E}_X\left[\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)\right]-\\&-\frac{e^{\beta_0'X}}{\alpha_0}\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)\ln\left(e^{-\beta_0'X}S+\varepsilon^{\alpha_0}\right)+\end{aligned} \tag{29}$$

$$+\mathrm{E}_X\left[\frac{e^{\beta_0' X}}{\alpha_0}\left(e^{-\beta_0' X}S+\varepsilon^{\alpha_0}\right)\ln\left(e^{-\beta_0' X}S+\varepsilon^{\alpha_0}\right)\right]+$$
$$+\frac{e^{\beta_0' X}\varepsilon^{\alpha_0}\ln\varepsilon}{S}-\mathrm{E}_X\left[\frac{e^{\beta_0' X}\varepsilon^{\alpha_0}\ln\varepsilon}{S}\right]$$

is constant in S for all x in the support of X, and this is true if and only if $\varepsilon = 0$.

Appendix C Proof of Proposition 2 if the number of parameters in the baseline hazard is greater than 1.

If α is a vector and $\alpha \in B(\alpha_0)$, (25) becomes

$$c(\alpha)'\frac{\partial \ln \lambda(t,\alpha)}{\partial \alpha} - c(\alpha)'\frac{\partial \ln \Lambda(t,\alpha)}{\partial \alpha} = f(\alpha) \tag{30}$$

for some vector $c(\alpha)$ and a function, $f(\alpha)$. Consider the case that α has two parameters. Then from (30)

$$\frac{\partial \ln \lambda(t,\alpha)}{\partial \alpha_1} - \frac{\partial \ln \Lambda(t,\alpha)}{\partial \alpha_1} = \frac{f(\alpha)}{c_1(\alpha)} - \frac{c_2(\alpha)}{c_1(\alpha)}\left(\frac{\partial \ln \lambda(t,\alpha)}{\partial \alpha_2} - \frac{\partial \ln \Lambda(t,\alpha)}{\partial \alpha_2}\right). \tag{31}$$

Integrating with respect to α_1 and and t yields the representation $\Lambda(t,\alpha) = h(t,\alpha)^{d(\alpha)}$ with $d(\alpha) = e^{\int_{\alpha_{10}}^{\alpha_1} \frac{f(\gamma,\alpha_2)}{c_1(\gamma,\alpha_2)} d\gamma}$ and $\ln h(t,\alpha_0) = \int_{t_0}^{t} e^{k(s,\alpha_2) - \int_{\alpha_{10}}^{\alpha_1} \frac{c_2(\gamma,\alpha_2)}{c_1(\gamma,\alpha_2)}\left(\frac{\partial \ln \lambda(s,\gamma,\alpha_2)}{\partial \alpha_2} - \frac{\partial \ln \Lambda(s,\gamma,\alpha_2)}{\partial \alpha_2}\right) d\gamma} \mathrm{d}s$ so that Proposition 2 still holds with an obvious modification.

Appendix D

Addendum: Duration dependence near 0 in empirical research

Structural/empirical papers not MPH

In some papers only a graph of the baseline hazard is provided, and no estimates (with standard errors).

1. Van den Berg, ReStud (1990). Unemployment durations. Hazard is $\theta(t,\alpha) = \lambda\overline{F}(\phi(t))$ with $\phi(t)$ the time-varying reservation wage. In his application $\phi(t)$ is bounded and the arrival rate and wage offer distribution are time constant. Hence the hazard near 0 is bounded from 0 and ∞.

2. Blau and Robins, J. of Public Economics (1986). Unemployment durations (days). Piecewise constant baseline hazard (exponential specification) no unobserved heterogeneity. Estimates of offer arrival rate (Tables 2-3) and re-employment hazard (Table 4). Baseline hazard in first 10 weeks not different.

3. Yoon, Economics Letters (1985). Derives closed form solution for hazard in non-stationary job search. This hazard satisfies the condition A2*.

Reduced form studies (not Weibull)

1. Arulampalam and Stewart, Econ Journal (1995). Unemployment durations. Meyer type grouped duration model with piecewise constant baseline hazard (weeks), no unobserved heterogeneity. Baseline hazard is in Figure 1, p.327. Greater than 0 and finite near 0.

2. Bonnal, Fougere, Serandon, ReStud (1997). Transitions between various states. Piecewise constant baseline hazard and unobserved heterogeneity. Intercept in Table 4. p.702 is baseline hazard near 0. Estimates consistent with A2*

3. Dolton and Van der Klaauw, Econ Journal (1995). Time to leaving the teaching profession. Meyer type estimation with unobserved heterogeneity. Baseline hazard near 0 is 0 in graph. See Fig 2 and 3, p. 439-440. A2* may be problematic.

4. Meyer, Ectra (1990). Unemployment durations (weeks). Grouped duration model with unobserved heterogeneity. Hazard in first week reported in Table VII, p. 774. Significantly different from 0. Inverse also (use delta method).

5. Gönül and Srinivasan, JASA (1993). Times between brand switches (two-state model) in weeks. Piecewise constant hazard with unobserved heterogeneity. Estimates with-

out (Table 2) and with (Table 4) unobserved heterogeneity. Consider estimate in first month. A2* OK.

6. Blank, J. of Public Economics (1989). Welfare spells in months. Piecewise constant baseline (exponential specification)hazard without unobserved heterogeneity. See Figure 4 for graph. A2* OK.

7. Ham and Rea, JOLE (1987). Unemployment durations (weeks). Discrete hazard (logit specification), not MPH. Bathtub shape probabilities of leaving unemployment. Empirical hazard in Table 1. Figures 1-4 give re-employment probability that is well below 1.

8. Kennan, JOEC (1985). Strike durations in days. Discrete hazard (logit specification). Probabilities reported in figures 1-6. Probability of settlement is not 0 on first day and this probability is not near 1 (and not the largest).

9. Flinn and Heckman, Adv. in Ectrics (1982). Two-state model with employment and non-employment. Spells in days? Baseline hazard is two-component Box-Cox with the λ's fixed at 1 and 2. Baseline hazard satisfies A2*.

10. Follain, Ondrich, and Sinha, Journal of Urban Economics (1997). Time to repayment of mortgage in quarters. Meyer type grouped duration data with unobserved heterogeneity. Hazard in first quarter much smaller than in later quarters. Figure 1 suggests that hazard starts at 0. A2* may be problematic.

References in addendum

Arulampalam, W., and M. Stewart (1995): "The determinants of individual unemployment durations in an era of high unemployment." *Economic Journal* 105, 321-32.

Blank, R. (1989): "Analyzing the length of welfare spells." *Journal of Public Economics* 39, 245-75.

Bonnal, L., F. Fougere, and A. Serandon (1997): "Evaluating the impact of French employment policies on individual labour market histories." *Review of Economic Studies* 64, 683-713.

Dolton, P., and W. Van der Klaauw (1995): "Leaving teaching in the UK: A duration analysis." *Economic Journal* 105, 431-44.

Follain, J. R., J. Ondrich, and G. P. Sinha (1997): "Ruthless prepayment? Evidence from multifamily mortgages." *Journal of Urban Economics* 41, 78-101.

Gönül, F., and K. Srinivasan. "Consumer purchase behavior in a frequently bought product category: Estimation issues and managerial insights from a hazard." *Journal of the American Statistical Association* 88, 1219-28.

Ham, J. C., and S. A. Rea (1987): "Unemployment insurance and male unemployment duration in Canada." *Journal of Labor Economics* 5, 325-53.

Neumann, G. R. (1997): "Search Models and Duration Data," Chapter 7 in *Handbook of Applied Econometrics: Microeconometrics*, M.H. Pesaran, ed., Basil Blackwell: Oxford, 300-351.