

Mortgage Default and Low Downpayment Loans:
The Costs of Public Subsidy*

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Abstract

This paper presents a unified model of the default and prepayment behavior of homeowners in a proportional hazard framework. The model uses the option-based approach to analyze default and prepayment, and considers these two interdependent hazards as competing risks.

The results indicate the sensitivity of default to the initial loan-to-value ratio of the loan and the course of housing equity. The latter is a measure of the extent to which the default option is in the money. The results also indicate the importance of trigger events, namely unemployment and divorce, in affecting prepayment and default behavior.

The empirical results are used to analyze the costs of a current policy proposal — stimulating homeownership by offering low downpayment loans. We simulate default probabilities and costs on zero-downpayment loans and compare them to conventional loans with conventional underwriting standards. The results indicate that if zero-downpayment loans were priced as if they were mortgages with ten percent downpayments, then the additional program costs would be two to four percent of funds made available — when housing prices increase steadily. If housing prices remained constant, the costs of the program would be much larger indeed. Our estimates suggest that additional program costs could be between \$74,000 and \$87,000 per million dollars of lending. If the expected losses from such a program were not priced at all, the losses from default alone could exceed ten percent of the funds made available for loans.

1. Introduction

This paper presents a unified model of the default and prepayment behavior of homeowners in a context which recognizes these hazards as competing risks. The model uses the option-based approach to analyze default and prepayment, that is, it views default as a put option and prepayment as a call option. The additional model also recognizes that the borrower, by exercising one of these two options, gives up the right to exercise the other option.

The model is applied to analyze a current policy proposal — transferring resources and stimulating homeownership by offering low downpayment loans. For instance, it has been proposed recently that the federal government, through FHA, insure zero downpayment mortgage loans for low income families.¹ We do not estimate the effectiveness of this policy at stimulating homeownership, but we do use our model to estimate program costs. We do this by comparing default probabilities and costs on zero downpayment loans to conventional loans requiring a five or ten percent downpayment. Obviously the subsidy inherent in zero downpayment loans compared to conventional loans depends on the pricing

¹Under the Clinton administration's fall 1994 initiative, mortgages would be targeted to first-time homebuyers with incomes below 115 percent of the median for the metropolitan area of residence. Mortgage amounts would be limited to the lesser of 75 percent of the FHA 203(b) loan limit for the location or 100 percent of property value.

of these loans. We can estimate subsidy costs directly if the difference in risk is not priced. FHA has, historically, not varied price significantly by downpayment.²

The starting point for option-based models is the contingent claims model, developed by Black and Scholes [3] and Cox, Ingersoll, and Ross [8]. A number of studies have applied this model to the mortgage market (e.g., Dunn and McConnell [12], Buser and Hendershott [5], Brennan and Schwartz [4], Kau et al [18][19], Quigley and Van Order [23]). Hendershott and Van Order [16], and Kau and Keenan [20] provide surveys of these models and results.

The key result of these models is that the exercise of **each** option depends on **both** house prices and interest rates. The default option is in the money when a borrower's equity is negative, and the prepayment option is in the money when the present value of the remaining payments is less when discounted at the current interest rate rather than the mortgage coupon rate. The contingent claims model describes how far into the money these options must be in order for it to be optimal for the borrower to exercise one of them. If there are any transactions costs, the mathematics of all this gets quite complicated. Empirical work exploiting the

²Prior to 1991, the FHA did not vary price by initial loan-to-value ratio (LTV). Currently all borrowers pay an up-front premium of 2.25 percent. They also pay one half of one percent of the loan balance each year for seven (in the case of LTV < 90 percent) to thirty (in the case of LTV > 95 percent) years. As indicated by our estimates below, the current variation does not nearly match the variation in risk.

option perspective has generally taken a probabilistic approach: each option is more likely to be exercised the further it is “in the money.”

Several recent empirical studies have applied the Cox proportional hazard model [7] to evaluate mortgage default or prepayment risk (e.g., Green and Shoven [15], Schwartz and Torous [25], Quigley and Van Order [22] [23]). Instead of solving for the unique critical values of the state variables in the contingent claims model, the proportional hazard model assumes that at each point in time during the mortgage contract period, the mortgage has a certain probability of termination, conditional upon the survival of the mortgage. The hazard function in this model is defined as the product of a baseline hazard and a function of time-varying covariates. These covariates need not be limited to the option value itself; they may include other important determinants of behavior. The proportional hazard model can thus incorporate reasonable mortgage prepayment and default behavior that would be considered “sub-optimal” under the pure contingent claims framework.

However, few of the existing empirical models have treated the interdependence of borrowers’ prepayment and default options. For example, Schwartz and Torous [25] analyzed GNMA mortgage prepayment experience by using a model with fixed

covariates³ and assuming the mortgages were free of default risk. More general models using time-varying covariates, by Green and Shoven [15] and Quigley and Van Order [22], made analogous assumptions in the analysis of prepayment behavior. Cunningham and Hendershott [10] analyze default costs in an optimizing context with transactions cost and a single hazard. Quigley and Van Order [23] studied default behavior using the model of a single hazard as well. Foster and Van Order [14] do estimate both default and prepayment, but they use highly aggregated data on mortgage pools.

2. The Model

In this paper we model the mortgage termination in a competing risks framework. The competing risks of mortgage termination consist of two parts: a prepayment risk and a default risk. The function specifying prepayment risk estimates the probability that a mortgage loan is prepaid during any period, conditional on survival to that particular period. Similarly, the default function estimates the conditional probability of default during each period. The model assumes that the borrower makes the prepayment or default decision based upon market conditions

³The option-related financial and economic variables relevant to mortgages clearly vary over time, so it would have been more appropriate to specify the model with time-varying covariates.

to maximize net wealth. Following the contingent claims model, we assume that the probability of exercising these options is a function of the extent to which they are “in the money” and of “trigger events” that affect the decision about how far the option need be into the money in order for exercise to be optimal. For instance, an increase in the probability of negative equity will increase the probability that the put option is in the money, and hence increase the probability of default. Examples of trigger events include such economic variables as employment or divorce.⁴

One particular feature of mortgage prepayment and default is the possibility of right censoring. There are two sources of censoring:

First, some mortgage loans may simply mature under the contract, and some mortgages may not terminate by the end of data collection period. For those mortgage loans, we simply never observe their actual durations to default or prepayment;

Second, and more importantly, if a mortgage loan has defaulted, it cannot be prepaid in the future. Thus defaulted loans are treated as censored data for the prepayment function, and prepaid loans are treated as censored data for the

⁴Job loss or household dissolution leads to reduced ability to fulfil monthly payment obligations and thus increases the likelihood of mortgage termination by default.

default functions.

Both of these mechanisms are random censoring. It is reasonable to assume the former random censoring mechanism (e.g., the maturation of loans) is independent of the default or prepayment failure time series. However, the latter mechanism need not be an independent random censoring mechanism at all. Indeed, for this reason, we should expect there to be a correlation between defaults and prepayments.⁵

2.1. A Semiparametric Estimation Approach for the Proportional Hazard Model

A major concern in actually estimating hazard models of mortgage prepayment and default behavior is the computational difficulty involved. Useful models must be specified in short time intervals (e.g., months or quarters), but mortgage terms are typically written for thirty years. Computational time can become a real constraint when the model involves time-varying covariates. To estimate a useful model for the housing market requires either dramatically limiting sample sizes, arbitrarily and unreasonably aggregating time intervals, or else finding a way to

⁵For example, households with negative equity may be more likely to default, but negative equity also makes it less likely that a household will choose to prepay.

aggregate observations on individual behavior.⁶

In this paper, we use a semiparametric estimation approach (SPE) to estimate the proportional hazard model with competing risks and time-varying covariates. This approach is described in detail elsewhere [11]; here we merely sketch out the major points.

Define $T \in R^+$ as a duration variable. Let T_i ($i = 1, 2, \dots, q$) be the discrete time intervals that partition the support of T . Let

$$h_j(t, Z) = h_{0j}(t) \left[\exp \left(Z_j(t) \beta_j \right) \right] \eta_j, \quad j = 1, 2 \quad (2.1)$$

be the conditional probability of a mortgage terminating at t . Here $j = 1$ is the prepayment function and $j = 2$ is the default function.

A log integrated hazard function for risk type j can be constructed:

$$\log \left[\int_{T_{i-1}}^{T_i} h_j(t, Z_j) dt \right] = Z_j(T_i) \beta_j + \gamma_j(T_i) + \varepsilon_j, \quad (2.2)$$

where

$$\gamma_j(T_i) = \log \left[\int_{T_{i-1}}^{T_i} h_{0j}(t) dt \right], \quad (2.3)$$

⁶See Deng, Quigley and Van Order [11], for a discussion of alternative models and computational methods.

and

$$\varepsilon_j = \log \eta_j,$$

$$j = 1, 2, \dots, J, \quad i = 1, 2, \dots, q,$$

given that $Z_j(t)$ is constant between T_{i-1} and T_i .

The left-hand side of equation (2.2) is not directly observable in micro data. We can, however, use the “local smoothing” technique, developed in the literature on non-parametric methods, to estimate individual hazard functions based on the empirical distribution of the hazard functions. Partition the covariate matrix Z into K distinct matrices Z_1, \dots, Z_K . The k th subgroup contains M_k observations. $M_1 + M_2 + \dots + M_K = N$, where N is the total sample size. For each subgroup, estimate the hazard rate such that $\hat{h}_{jkt} = \frac{n_{jkt}}{S_{kt}}$, where n_{jkt} is the number of loans which are terminated in the t th period with type j in the k th subgroup, and S_{kt} is the total number of loans surviving to the t th period in the k th subgroup.⁷

Now, replacing the left-hand side of equation (2.2) with the smoothed log

⁷Note the risk set of the conditional hazard rates includes not only the loans that have the same termination type, but also all those loans which have a different termination type, as long as the age of termination is greater than the current one. Furthermore, the risk set also includes those right-censored observations if the censored time point is greater than the current termination time.

hazard function, $\log \int_{T_{i-1}}^{T_i} \widehat{h}_{jk}(t, Z_{jk}) dt$, yields

$$\log \left[\int_{T_{i-1}}^{T_i} \widehat{h}_{jk}(t, Z_{jk}) dt \right] = Z_{jk}(T_i) \beta_j + \gamma_j(T_i) + \varepsilon_j + u_{jk}(T_i), \quad (2.4)$$

$$j = 1, 2, \dots, J, \quad k = 1, 2, \dots, K, \quad i = 1, 2, \dots, q,$$

where $u_{jk}(T_i) = \log \left[\int_{T_{i-1}}^{T_i} \widehat{h}_{jk}(t, Z_{jk}) dt \right] - \log \left[\int_{T_{i-1}}^{T_i} h_{jk}(t, Z_{jk}) dt \right]$.

For a model in which time is measured in discrete intervals, say, quarters, equation (2.4) reduces to

$$\log \left[\widehat{h}_{jk}(T_i, Z_{jk}) \right] = Z_{jk}(T_i) \beta_j + \gamma_j(T_i) + \varepsilon_j + u_{jk}(T_i). \quad (2.5)$$

The covariance of the ε_j 's captures the correlation among competing risks. Equations (2.4) and (2.5) are seemingly unrelated regression systems that can be analyzed using the approach proposed by Zellner [28]. It has been shown elsewhere [11] that the coefficient vector $\widehat{\beta}$ estimated from equation (2.4) is consistent.

3. The Empirical Analysis

3.1. The Data

The empirical analysis is based upon individual mortgage history data maintained by the Federal Home Loan Mortgage Corporation (Freddie Mac). The data base contains 1,489,372 observations on single family mortgage loans issued between 1976 to 1983 and purchased by Freddie Mac. All are fixed-rate, level-payment, fully-amortized loans, most of them with thirty-year terms. The mortgage history period ends in first quarter of 1992. For each mortgage loan, the available information includes the year and month of origination and termination (if it has been closed), indicators of prepayment or default, the purchase price of the property, the original loan amount, the initial loan-to-value ratio, the mortgage contract interest rate, the monthly principal and interest payment, the state, the region and the major metropolitan area in which the property is located. The data set also reports the borrower's monthly gross income at loan origination. For the mortgage default and prepayment model, censored observations include all matured loans as well as the loans active at the end of the period.

The analysis is confined to 30 year fixed rate mortgage loans issued for owner occupancy, and includes only those loans which were either closed or still active

at the first quarter of 1992.⁸ The analysis is confined to loans issued in 26 major metropolitan areas (MSAs). The data set contains 780,443 observations. Loans are observed in each quarter from the quarter of origination through the quarter of termination, maturation, or through 1992:I for loans still active.

To estimate the model, the entire sample of 780,443 loans has been partitioned into 312 groups, according to 26 major MSAs, 4 household income levels, and 3 LTV groups.⁹ For each group, there are 64 cells, reflecting failure time periods (measured in quarter, from 76:II to 92:I). We use the Kaplan-Meier approach to fit the empirical hazard rates of prepayment and default based on the entire sample. Then the estimated empirical hazard rates were mapped to 11,866 mortgage loans that were randomly drawn from the total sample.¹⁰ We assume that the randomly-drawn subsample has the same distribution as the population.

The appendix provides some descriptive statistics of the subsample of 11,866 observations used in the regression. A majority of the mortgage loans have an

⁸It excludes those observations which were in delinquency or foreclosure at the time data were collected.

⁹The 3 LTV groups are: LTV less than or equal to 0.8, LTV greater than 0.8 and less than 0.95, and LTV greater than or equal to 0.95.

¹⁰We confine the random drawing within the subsample of those cells that have non-zero empirical hazard rates. It should be pointed out that if the cell-partition is too small, then it might increase the occurrence of zero empirical hazard rates, especially for default hazard rates, for certain cells. Consequently, the drawing will over sample default loans by eliminating those loans assigned with zero empirical default hazard rates. (To obtain an optimal partition size, we could have used the cross-validation technique, which is familiar in literature on nonparametric estimation.)

equity ratio greater than 0.3; only 1.6 percent of loans have an equity ratio less than 0.1. About 2.2 percent of the households had incomes below 60 percent of the MSA median at loan origination, and 43 percent of the households had incomes above 150 percent of the MSA median. About 12 percent of the loans were issued with a loan to value ratio (LTV) below 0.6, and 15 percent of the loans had an initial LTV at or above 0.95.

Figures 1 and 2 summarize the raw data on mortgage terminations which underlie the empirical analysis. Figure 1 displays the conditional prepayment rate, separately by loan-to-value ratio (LTV), as a function of duration. Conditional prepayment rates are slightly higher for higher LTV loans. Rates increase substantially after the first fifteen quarters. Figure 2 displays raw conditional default rates by LTV. Note again that default rates increase substantially after about fifteen quarters. Note also that the default rates increase very substantially with initial LTV. Default rates for loans with LTV above 95 percent are three or four times higher than default rates for 90 to 95 percent LTV loans. The default rates for these latter loans are, in turn, about five times as high as for those with LTV below 80 percent.

Finally, note that conditional default rates are quite low. Even for the riskiest class of loans, conditional default rates are no higher than four in a thousand per

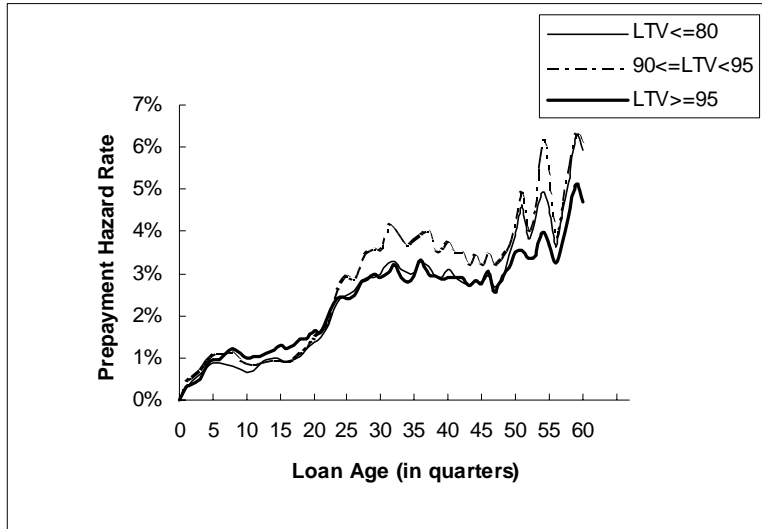


Figure 1: Conditional Prepayment Rates by LTV

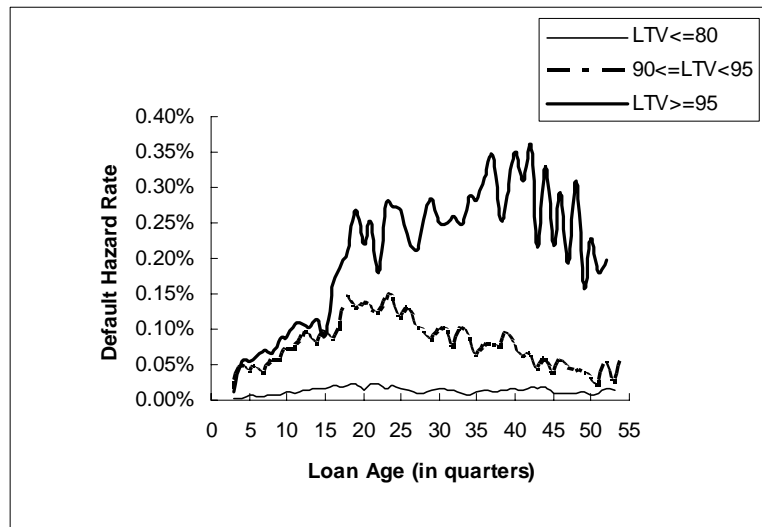


Figure 2: Conditional Default Rates by LTV

quarter. Residential mortgages are relatively safe investments (and simple random samples of mortgages are likely to contain very few observations on default).

The key variables in the theoretical model are those measuring the extent to which the put and call options are in the money. To value the call option, the current interest rate, and the initial contract terms are sufficient. We compute a variable "*POPTION*" measuring the ratio of the present discounted value of the unpaid mortgage balance at the current quarterly market interest rate¹¹ relative to the value discounted at the contract interest rate.¹²

To value the put option analogously, we need to measure the market value of each house quarterly and to compute homeowner equity quarterly. Obviously, we

¹¹The rate used is the average interest rate charged by lenders on new first mortgages reported by Freddie Mac's quarterly market survey (the "effective Freddie rate"). This mortgage interest rate varies by quarter across five major US regions.

¹²Specifically, *poption* for the l th loan observation is defined as

$$\begin{aligned}
 poption_l &= \frac{\sum_{t=1}^{term_l - \tau_l} \frac{mopipmt_l \times 3}{(1 + mktrate_{\omega_l, \kappa_l + \tau_l} / 400)^t} - \sum_{t=1}^{term_l - \tau_l} \frac{mopipmt_l \times 3}{(1 + noterate_l / 400)^t}}{\sum_{t=1}^{term_l - \tau_l} \frac{mopipmt_l \times 3}{(1 + mktrate_{\omega_l, \kappa_l + \tau_l} / 400)^t}} \\
 &= 1 - \frac{mktrate_{\omega_l, \kappa_l + \tau_l} \times \left(1 - \left(\frac{1}{1 + noterate_l / 400}\right)^{term_l - \tau_l}\right)}{noterate_l \times \left(1 - \left(\frac{1}{1 + mktrate_{\omega_l, \kappa_l + \tau_l} / 400}\right)^{term_l - \tau_l}\right)},
 \end{aligned} \tag{3.1}$$

where τ_l is loan age measured in quarters, ω_l is a vector of indices for geographical location, κ_l is loan origination time, $mopipmt_l$ is monthly principal and interest payment, $noterate_l$ is mortgage note rate, $mktrate_{\omega_l, \kappa_l + \tau_l}$, is the current local market interest rate, and $term_l$ is

do not observe the course of price variation for individual houses in the sample. We do, however, have access to a large sample of repeat (or paired) sales of single family houses in 26 metropolitan areas (MSAs). This information is sufficient to estimate a weighted repeat sales housing price index (WRS) separately for each of the 26 MSAs. The WRS index provides estimates of the course of housing prices in each metropolitan area. It also provides an estimate of the variance in price for each house in the sample, by metropolitan area and elapsed time since purchase.¹³

mortgage loan term calculated by

$$term_l = \frac{\log\left(\frac{mopipmt_l - origamt_l \times (noterate_l/1200)}{mopipmt_l}\right)}{\log\left(\frac{1}{1 + noterate_l/1200}\right) \times 3}, \quad (3.2)$$

where $origamt_l$ is original loan amount.

¹³Housing price indices and their volatilities are estimated according to the three stage procedure suggested by Case and Shiller [6] and modified by Quigley and Van Order[23]. The model assumes that log price for i th house at time t is given by

$$P_{it} = I_t + H_{it} + N_{it} \quad (3.3)$$

where I_t is the logarithm of the regional housing price level, H_{it} is a Gaussian random walk, such that,

$$\begin{aligned} E[H_{i,t+\tau} - H_{it}] &= 0, \\ E[H_{i,t+\tau} - H_{it}]^2 &= \tau\sigma_{\eta_1}^2 + \tau^2\sigma_{\eta_2}^2; \end{aligned}$$

and N_{it} is white noise, such that,

$$\begin{aligned} E[N_{it}] &= 0, \\ E[N_{it}]^2 &= \frac{1}{2}\sigma_v^2. \end{aligned}$$

The model is estimated on paired sales of owner occupied housing. In the first stage, the log price of the second sale minus the log price of the first sale is regressed on a set of dummy variables, one for each time period in the sample except the first period. The dummy variables have values of zero in every quarter except the quarter in which the sales occurred. For the quarter of the first sale, the dummy is -1 , and for the quarter of the second sale, the dummy is

Estimates of the mean and variance of individual house prices, together with the unpaid mortgage balance (computed from the contract terms), permit us to estimate the distribution of homeowner equity quarterly for each observation. In particular, "EQR" is the estimate of equity ratio assuming prices of all houses in the MSA grow at the mean rate, "PNEQ" is the probability that equity ratio is negative, i.e., the probability that the put option is in the money.¹⁴

+1. (This follows Bailey, Muth, and Nourse [2] exactly.)

In the second stage, the squared residuals (e^2) from each observation in the first stage are regressed upon τ and τ^2

$$e^2 = A + B\tau + C\tau^2, \quad (3.4)$$

where τ is the interval between the first and second sale. The coefficients A , B , and C are estimates of σ_ν^2 , $\sigma_{\eta_1}^2$, and $\sigma_{\eta_2}^2$ respectively.

In the third stage, the stage one regression is reestimated by GLS with weights $\sqrt{A + B\tau + C\tau^2}$.

The estimated log price level difference ($\widehat{I}_{t+\tau} - \widehat{I}_t$) is normally distributed with mean ($I_{t+\tau} - I_t$), and variance ($\tau\sigma_{\eta_1}^2 + \tau^2\sigma_{\eta_2}^2 + \sigma_\nu^2$). Denote $msa_\tau = \exp(\widehat{I}_\tau)$ as the estimated regional housing price index; then $\log\left(\frac{msa_{\kappa+\tau}}{msa_\kappa}\right)$ is normally distributed with mean ($I_{\kappa+\tau} - I_\kappa$) and variance ($\tau\sigma_{\eta_1}^2 + \tau^2\sigma_{\eta_2}^2 + \sigma_\nu^2$).

Means and Variances are estimated for each of 26 major MSA regions using samples of paired sales. There are about four million paired sales in the Freddie Mac data base.

¹⁴Specifically, equity ratio for the l th loan observation is defined as:

$$\begin{aligned} eqr_l &= \frac{mktvalue_l - pdvunplc_l}{mktvalue_l} \\ &= \frac{purprice_l \times \frac{msa_{\omega_l, \kappa_l + \tau_l}}{msa_{\omega_l, \kappa_l}} - \sum_{t=1}^{term_l - \tau_l} \frac{mopipmt_l \times 3}{(1 + noterate_l/400)^t}}{purprice_l \times \frac{msa_{\omega_l, \kappa_l + \tau_l}}{msa_{\omega_l, \kappa_l}}} \\ &= 1 - \frac{(LTV/100) \times \left(1 - \left(\frac{1}{1 + noterate_l/400}\right)^{term_l - \tau_l}\right)}{\left(\frac{msa_{\omega_l, \kappa_l + \tau_l}}{msa_{\omega_l, \kappa_l}}\right) \times \left(1 - \left(\frac{1}{1 + noterate_l/400}\right)^{term_l}\right)}, \end{aligned} \quad (3.5)$$

For each mortgage loan observation, we calculate the ratio of household reported income (at origination) to the MSA median income level. We then create four dummy variables, "*INCLL*", "*INCL*", "*INCH*", and "*INCHH*" to separate household income level into four groups.¹⁵

As proxies for other "trigger events," we include measures of the quarterly unemployment rate and the annual divorce rate.¹⁶ Unfortunately, we do not have access to borrower's credit history, which might be important in determining default. Hence, we assume that the model has heterogeneous error terms due to these omitted variables.

where $purprice_t$ is the purchasing price of the house at the time of loan initiation, and $pdvunpblc_t$ is the present discounted value of the remaining loan balance.

The probability of negative equity, $pneq$, is thus

$$pneq_t = \text{ncdf} \left(\frac{\log(pdvunpblc_t) - \log(mktvalue_t)}{\sqrt{e_{\omega_1, \kappa_1 + \tau_1}^2}} \right), \quad (3.6)$$

where $pdvunpblc_t$ and $mktvalue_t$ are defined above, $\text{ncdf}(\cdot)$ is cumulative standard normal distribution function, and $e_{\omega_1, \kappa_1 + \tau_1}^2$ is the estimated volatility of the housing price index using the WRS procedure.

¹⁵Specifically, *incll* takes value one if the ratio of the household reported income to the MSA median level is less than or equal to 0.6, and zero otherwise; *incl* takes value one if the ratio lies between 0.6 and 1.0, and zero otherwise; *inch* takes value one if the ratio lies between 1.0 and 1.5, and zero otherwise; and *inchh* takes value one if the ratio is above 1.5, and zero otherwise.

¹⁶Unemployment and divorce rates are measured at the state level. State unemployment data are reported in various issues of: US Department of Labor, "Employment and Unemployment in States and Local Areas (Monthly)" and in the "Monthly Labor Review". State divorce data are reported in various issues of U.S. National Center for Health Statistics, "Vital Statistics of the United States, Volume III, Marriage and Divorce", and in "Statistical Abstract of the U.S."

3.2. The Empirical Results

Table 1 presents a variety of models estimated by the SPE method, specifying the prepayment and default functions as a seemingly unrelated regression system.

The results from all four models show that financial motivation is of paramount importance in affecting the prepayment and default behavior. When the call option is in the money, the prepayment hazard increases substantially. Similarly, a higher probability of negative equity increases the default hazard substantially for all income groups. Note that a higher probability of negative equity also reduces the prepayment hazard, reflecting the negative relationship between the values of these two options.

The coefficients estimated for the interaction between household income level and the equity variables from models 3 and 4 convey consistent information. Lower income households are more sensitive to lower equity values when making decisions to exercise the put option — that is, lower income households are at greater risk for default than high income households when equity values decline. This is true for each comparison if we exclude the highest income households. The results show that the very wealthy households (i.e., those with a ratio of household income to the MSA median income above 1.5) are apparently more likely to behave in a ruthless fashion in the face of equity declines.

The results are consistent with simple tabulations indicating that both low income and high income borrowers tend to default more than borrowers with moderate incomes (e.g., Van Order and Schnare [26].) However, the results also reveal a pattern of different responses toward declining equity among different income groups.

The results from all four models show that higher unemployment and divorce rates will lower the prepayment hazard – indicating that liquidity constraints (which make refinancing more difficult for unemployed and divorced households) keep them from exercising in-the-money call options. However, the coefficient estimates for these two variables are less stable in the default function.¹⁷

For each of the models reported in table 1, we have also estimated baseline hazard for prepayment and default; this estimation is discussed in detail in Quigley, Van Order and Deng [24]. We now apply these models and the estimated baselines to simulate effects of low downpayment loans.

¹⁷These two variables are measured at the state level. Thus, it would be an understatement to observe that they are prone to measurement error.

4. Simulations of Default Losses and Program Costs

The simulations of default losses are based on model 4 in Table 1. We use Freddie Mac's existing simulation model to generate 300 paths of mortgage market rates and 300 paths of house price inflation rates according a joint stochastic mean-reverting process. We consider three alternative patterns of housing price change: average annual appreciation rates of five and ten percent, and a benchmark case of zero percent change, on average. We fix the divorce rate at the mean and vary the aggregate unemployment rate between four percent and eight percent. For each scenario, we calculate 10,000 default hazard rates for zero downpayment loans, five percent and ten percent downpayment loans for the first 15 years of the mortgage loan period. We do this by the repeated sampling of housing prices and interest rates from the joint distributions and the calculation of conditional default and prepayment probabilities for each draw.

Table 2 presents estimates of the average default rates associated with these economic conditions. The table presents the default rate, cumulated over fifteen years, for zero downpayment loans and also for conventional mortgage loans covering 90 and 95 percent, respectively, of house purchase. Not surprisingly, the probability of ever defaulting is sensitive to variations in housing prices and also

to the aggregate unemployment rate. Default probabilities are also quite sensitive to the required downpayment as well as household income level. Consider “lower income home purchases,” i.e., households with incomes below 60 percent of the MSA median level. The simulations suggest that, with zero down payment loans, these households would have cumulative default rates about twice as high as those whose mortgages require ten percent down – when house prices appreciate at 10 percent annually and the unemployment rate is 8 percent. With zero downpayment loans, these households would have cumulative default rates about four times as high as those whose mortgages require ten percent down – when house price levels are constant.

Figures 3 to 6 summarize the predicted loss severities associated with these mortgage loans, as well as the timing of those losses. We use the loss severities, as a fraction of mortgage balances and as a function of initial LTV, reported by Van Order and Zorn [27]. These severities, together with the time path of (unconditional) defaults, yield the time paths of losses.

Table 3 summarizes the estimated costs of the subsidy program. The table reports the present value of the losses over a fifteen year period, using an interest rate of ten percent. The first column of Table 3 reports the appropriate price, *ex post*, for mortgage loans with ten percent downpayments made under each

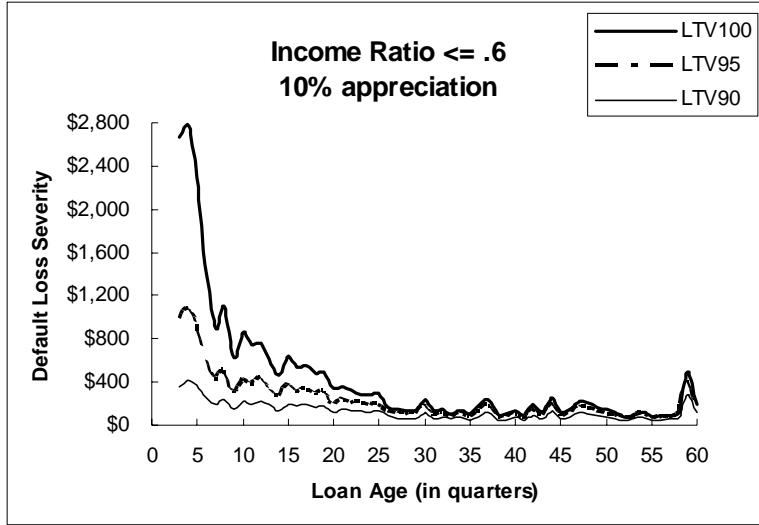


Figure 3: Simulated Time Path of Present Values of Default Losses

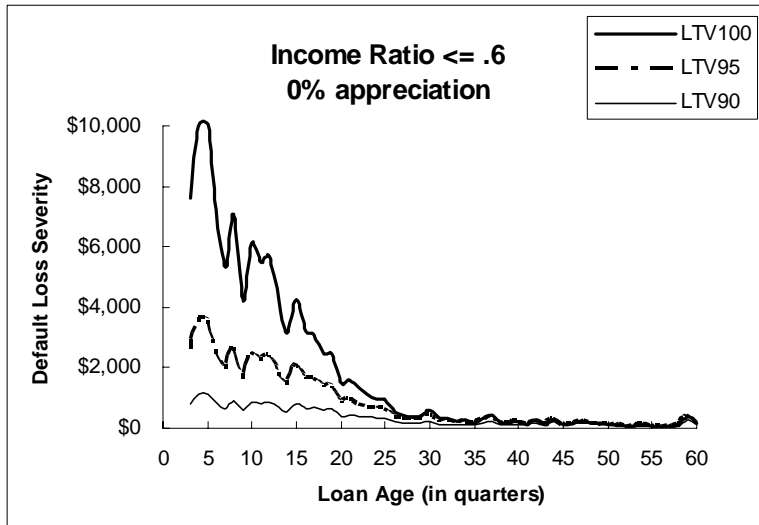


Figure 4: Simulated Time Path of Present Values of Default Losses

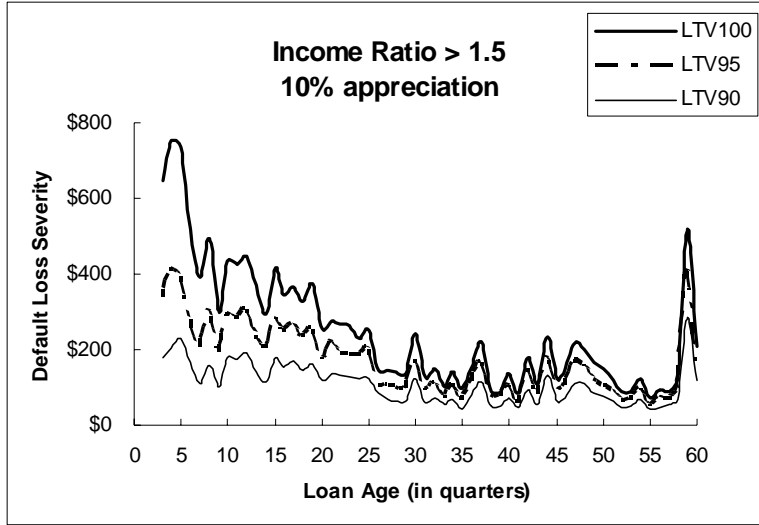


Figure 5: Simulated Time Path of Present Values of Default Losses

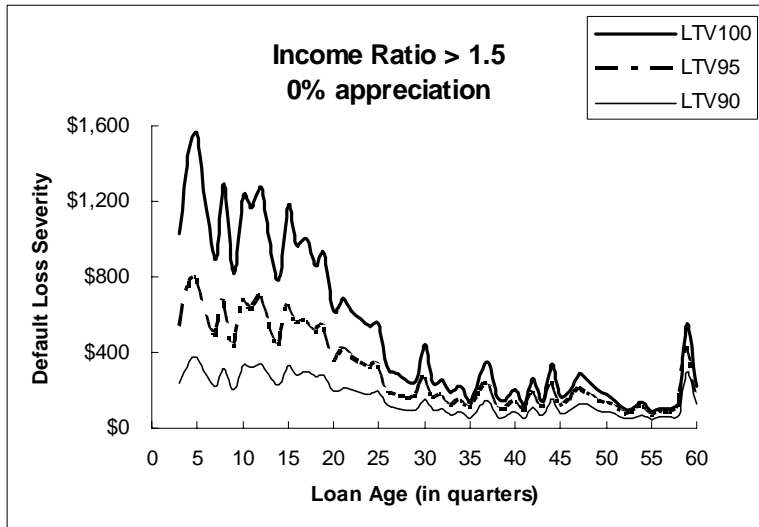


Figure 6: Simulated Time Path of Present Values of Default Losses

of the set of economic conditions described. For example, for a borrower who has household income at 60 percent of the MSA median income, the appropriate risk premium for 90 percent LTV loans, under circumstances in which there is ten percent increase in housing prices annually, is 0.74 percent. If we assume that the cost to consumers of zero downpayment mortgage loans is the same as appropriately priced ten-percent-down mortgage loans, the unpriced subsidy is about 1.67 percent.

The costs are quite large if house prices do not appreciate. According to table 3, if the unemployment rate is 8 percent, then for loans to households with incomes below 60 percent of the MSA median, the expected loss from a \$50,000 mortgage issued is \$5,332. If these mortgages are priced to consumers as if they were ten-percent-down loans, the cost to the U.S. Treasury would be \$4,356 for each \$50,000 mortgage originated.

5. Conclusions

This paper presents a competing risks model of mortgage terminations and uses the model to analyze the cost of a currently proposed housing subsidy policy, namely a policy of zero downpayment mortgage loans for selected households. The empirical model estimates jointly the competing risks of mortgage default

and prepayment in a multiple hazard framework with time-varying covariates.

The model indicates the sensitivity of default to the initial loan-to-value ratio of the loan and the course of housing equity. The latter is a measure of the extent to which the default option is in the money.

The empirical model is used to analyze the subsidy provided, and the program costs, of zero downpayment mortgages. The cost analysis is quite conservative, in that it assumes that the recipients of zero downpayment mortgages differ from households making substantial down payments only in the time path of their housing equity.

The actual program costs depend upon the pricing of these mortgages to consumers. If they were priced in a manner appropriate to mortgages with ten percent downpayments, the additional estimated program costs are around two to four percent of funds made available — when housing prices increase steadily.

In an economy where the house prices do not appreciate, the costs of the program would be much larger indeed. Our estimates suggest that additional program costs would be between \$74,000 and \$87,000 per million dollars of lending, for borrowers with incomes below 60 percent of the MSA median. If the expected losses from the program were not priced at all, the losses from default could exceed ten percent of the funds available for loans.

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Table 1. Estimates of SPE for Mortgage Prepayment and Default

	Model 1		Model 2		Model 3		Model 4	
	Prepmt	Default	Prepmt	Default	Prepmt	Default	Prepmt	Default
POPTION	0.145 (7.52)		0.152 (7.82)		0.190 (10.28)		0.196 (10.57)	
PNEQ	-2.179 (49.66)	4.55 (34.52)	-2.229 (49.54)	3.497 (27.45)				
INCOME	0.065 (17.57)	-0.138 (12.67)	0.066 (17.73)	-0.121 (11.76)				
PNEQ × INCLL					-0.639 (2.04)	9.154 (9.42)	-0.681 (2.18)	8.197 (8.94)
PNEQ × INCL					-3.567 (54.42)	5.692 (27.97)	-3.619 (54.50)	4.501 (23.15)
PNEQ × INCH					-1.862 (30.28)	3.388 (17.81)	-1.902 (30.66)	2.503 (13.84)
PNEQ × INCHH					-0.786 (8.86)	5.393 (19.71)	-0.844 (9.44)	4.091 (15.72)
LTV			0.115 (5.05)	2.477 (38.17)			0.105 (4.73)	2.491 (38.33)
UNEMPLOY	-0.067 (28.90)	-0.008 (1.10)	-0.066 (28.08)	0.036 (5.42)	-0.070 (30.92)	0.019 (2.77)	-0.069 (30.14)	0.046 (7.02)
DIVORCE	-0.126 (23.01)	-0.021 (1.30)	-0.126 (23.01)	-0.024 (1.57)	-0.116 (21.82)	-0.046 (2.77)	-0.116 (21.79)	-0.045 (2.91)
Var of Residual	0.132	1.221	0.132	1.087	0.127	1.227	0.127	1.092
Cov of Residual	0.016		0.009		0.011		0.006	
R²	0.633	0.229	0.634	0.314	0.647	0.225	0.648	0.312

Note: All models estimated by seemingly unrelated regressions using observations on 11,866 mortgages. t ratios in parentheses.

Table 2. Average Cumulative Default After Fifteen Years

Unemployment Rate	Average Annual House Price Change	Household Income at Initiation to MSA Median Income	Down Payment Rate		
			Ten percent	Five percent	Zero percent
Eight Percent	10 percent	Ratio \leq 0.6	4.65%	6.25%	9.40%
		0.6 < Ratio \leq 1.0	4.34	5.31	6.71
		1 < Ratio \leq 1.5	4.08	4.79	5.70
		Ratio > 1.5	4.16	4.98	6.14
	5 percent	Ratio \leq 0.6	5.80	9.07	16.18
		0.6 < Ratio \leq 1.0	4.89	6.35	8.62
		1 < Ratio \leq 1.5	4.35	5.25	6.48
		Ratio > 1.5	4.54	5.72	7.50
	0 percent	Ratio \leq 0.6	9.12	17.35	34.82
		0.6 < Ratio \leq 1.0	6.15	8.72	12.88
		1 < Ratio \leq 1.5	4.90	6.20	8.00
		Ratio > 1.5	5.42	7.39	10.51
Four Percent	10 percent	Ratio \leq 0.6	3.33	4.60	7.17
		0.6 < Ratio \leq 1.0	3.09	3.84	4.95
		1 < Ratio \leq 1.5	2.87	3.40	4.09
		Ratio > 1.5	2.92	3.54	4.44
	5 percent	Ratio \leq 0.6	4.28	6.94	12.87
		0.6 < Ratio \leq 1.0	3.56	4.72	6.58
		1 < Ratio \leq 1.5	3.09	3.79	4.74
		Ratio > 1.5	3.24	4.15	5.57
	0 percent	Ratio \leq 0.6	7.02	13.88	28.96
		0.6 < Ratio \leq 1.0	4.62	6.73	10.22
		1 < Ratio \leq 1.5	3.56	4.58	6.03
		Ratio > 1.5	3.96	5.53	8.06

Note: Estimates are based on Model 4 of Table 1. Each rate is based upon 10,000 replications.

Table 3. Present Value of Losses per Million Dollars of Program Payments

Unemployment Rate	Average Annual House Price Change	Household Income at Initiation to MSA Median Income	Down Payment Rate		
			Ten percent	Five percent	Zero percent
Eight Percent	10 percent	Ratio \leq 0.6	\$7,424	\$13,524	\$25,107
		0.6 < Ratio \leq 1.0	6,600	10,504	15,796
		1 < Ratio \leq 1.5	6,030	9,224	12,675
		Ratio > 1.5	6,278	9,902	14,424
	5 percent	Ratio \leq 0.6	10,372	22,554	48,288
		0.6 < Ratio \leq 1.0	7,779	13,574	22,420
		1 < Ratio \leq 1.5	6,626	10,369	15,134
		Ratio > 1.5	7,194	12,188	19,062
	0 percent	Ratio \leq 0.6	19,507	48,020	106,629
		0.6 < Ratio \leq 1.0	10,947	21,101	37,131
		1 < Ratio \leq 1.5	7,848	13,040	19,798
		Ratio > 1.5	9,424	17,438	29,738
Four Percent	10 percent	Ratio \leq 0.6	5,575	10,484	19,759
		0.6 < Ratio \leq 1.0	4,960	8,032	12,307
		1 < Ratio \leq 1.5	4,500	6,844	9,473
		Ratio > 1.5	4,630	7,313	10,953
	5 percent	Ratio \leq 0.6	7,974	17,811	39,069
		0.6 < Ratio \leq 1.0	5,967	10,765	17,802
		1 < Ratio \leq 1.5	4,996	7,854	11,531
		Ratio > 1.5	5,414	9,141	14,838
	0 percent	Ratio \leq 0.6	15,442	39,392	89,814
		0.6 < Ratio \leq 1.0	8,542	16,872	30,368
		1 < Ratio \leq 1.5	5,924	10,085	15,512
		Ratio > 1.5	7,203	13,452	23,486

Appendix: Descriptive Statistics of the Sample of Mortgage Loans

Table A1. Frequency Distribution for Age of Loan at Termination

Termination Age	Frequency	Percent	Cumulative Frequency	Cumulative Percent
2	9	0.1	9	0.1
3	39	0.3	48	0.4
4	85	0.7	133	1.1
5	124	1.0	257	2.2
6	142	1.2	399	3.4
7	143	1.2	542	4.6
8	164	1.4	706	5.9
9	183	1.5	889	7.5
10	139	1.2	1,028	8.7
11	144	1.2	1,172	9.9
12	178	1.5	1,350	11.4
13	243	2.0	1,593	13.4
14	257	2.2	1,850	15.6
15	148	1.2	1,998	16.8
16	220	1.9	2,218	18.7
17	216	1.8	2,434	20.5
18	293	2.5	2,727	23.0
19	287	2.4	3,014	25.4
20	348	2.9	3,362	28.3
21	325	2.7	3,687	31.1
22	340	2.9	4,027	33.9
23	488	4.1	4,515	38.0
24	532	4.5	5,047	42.5
25	490	4.1	5,537	46.7
26	465	3.9	6,002	50.6
27	457	3.9	6,459	54.4
28	469	4.0	6,928	58.4
29	419	3.5	7,347	61.9
30	326	2.7	7,673	64.7
31	323	2.7	7,996	67.4
32	272	2.3	8,268	69.7
33	353	3.0	8,621	72.7
34	236	2.0	8,857	74.6
35	296	2.5	9,153	77.1
36	226	1.9	9,379	79.0
37	123	1.0	9,502	80.1
38	163	1.4	9,665	81.5
39	188	1.6	9,853	83.0
40	177	1.5	10,030	84.5
41	171	1.4	10,201	86.0
42	201	1.7	10,402	87.7
43	113	1.0	10,515	88.6
44	112	0.9	10,627	89.6
45	81	0.7	10,708	90.2

Table A1. Frequency Distribution for Age of Loan at Termination (continued)

Termination Age	Frequency	Percent	Cumulative Frequency	Cumulative Percent
46	101	0.9	10,809	91.1
47	52	0.4	10,861	91.5
48	92	0.8	10,953	92.3
49	100	0.8	11,053	93.1
50	103	0.9	11,156	94.0
51	71	0.6	11,227	94.6
52	251	2.1	11,478	96.7
53	214	1.8	11,692	98.5
54	45	0.4	11,737	98.9
55	24	0.2	11,761	99.1
56	30	0.3	11,791	99.4
57	45	0.4	11,836	99.7
58	5	0.0	11,841	99.8
59	2	0.0	11,843	99.8
60	23	0.2	11,866	100.0

Table A2. Distribution of Loan Terminations

Type of Termination	Frequency	Percent	Cumulative Frequency	Cumulative Percent
A: Termination				
Default	539	4.5	539	4.5
Prepayment	10,427	87.9	10,966	92.4
B: Censored	900	7.6	11,866	100.0

Table A3. Distribution of Estimated Equity Ratios at Termination

Equity Ratio	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Equity Ratio \leq 0.1	195	1.6	195	1.6
0.1 < Equity Ratio \leq 0.2	500	4.2	695	5.9
0.2 < Equity Ratio \leq 0.3	1,516	12.8	2,211	18.6
0.3 < Equity Ratio \leq 0.4	2,414	20.3	4,625	39.0
0.4 < Equity Ratio \leq 0.5	2,687	22.6	7,312	61.6
Equity Ratio > 0.5	4,554	38.4	11,866	100.0

Table A4. Distribution of Borrower Income at Origin Relative to MSA Median

Ratio of Household Income to MSA Median	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Ratio ≤ 60%	258	2.2	258	2.2
60% < Ratio ≤ 100%	2,332	19.7	2,590	21.8
100% < Ratio ≤ 150%	4,194	35.3	6,784	57.2
120% < Ratio ≤ 200%	2,303	19.4	9,087	76.6
Ratio > 200%	2,779	23.4	11,866	100.0

Table A5. Distribution of Loan to Value Ratio at Origin

LTV	Frequency	Percent	Cumulative Frequency	Cumulative Percent
LTV ≤ 60	1,383	11.7	1,383	11.7
60 < LTV ≤ 80	5,293	44.6	6,676	56.3
80 < LTV < 90	572	4.8	7,248	61.1
90 ≤ LTV < 95	2,831	23.9	10,079	84.9
95 ≤ LTV ≤ 100	1,787	15.1	11,866	100.0

Table A6. Distribution of Year of Origin of Mortgage

YR ORIGIN	Frequency	Percent	Cumulative Frequency	Cumulative Percent
1976	639	5.4	639	5.4
1977	1,195	10.1	1,834	15.5
1978	2,267	19.1	4,101	34.6
1979	2,704	22.8	6,805	57.3
1980	2,912	24.5	9,717	81.9
1981	1,127	9.5	10,844	91.4
1982	682	5.7	11,526	97.1
1983	340	2.9	11,866	100.0